VOLUME XXX NUMBER 5

THE PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

CONDUCTED
WITH THE CO-OPERATION OF THE

AMERICAN PHYSICAL SOCIETY

BY

EDWARD L. NICHOLS

ERNEST MERRITT, and FREDERICK BEDELL

CLXIX

MAY, 1910

The Terminal Velocity of Fall of Small Spheres in Air.
JOHN ZELENY and L. W. McKEEHAN 535

A Study of the Multiple Reflections of Short Electric Waves between two or more Reflecting Surfaces. L. E. WOODMAN and H. W. WEBB 561

The Absorption of Sodium Vapor. GEO. B. CLINKSCALES 594

The Elastic Properties of Platinum-Iridium Wires.
K. E. GUTHE and L. P. SIEG 619

"Bound Mass" and the Fitzgerald Lorentz Contraction. WILL C. BAKER 634

The Secondary Radiation Produced by the Beta Rays of Radium. W. H. BRAGG 638

A Convenient Form of Quartz Tube Mercury Lamp. CHAS. T. KNIPP 641

Note on the Reflecting Power of Tantalum, Tungsten, and Molybdenum.
W. W. COBLENTZ 645

The Cathode Equilibrium in the Weston Standard Cell. G. A. HULETT 648

Proceedings of the American Physical Society 656

Minutes of the Fifty-second Meeting. The Use of the Electrical Resistance Thermometer in Apparatus for the Absolute Measurement of Solar Radiation, *C. F. Marvin*; Note on the Relation Between the Resistance of Nickel and the Temperature, *C. F. Marvin*; The Effect of Filter Paper upon the Mass and Form of the Deposit in the Silver Coulometer, *E. B. Rosa, G. W. Vinal and A. S. McDaniel*; Some Curious Phenomena Observed in Connection with Melde's Experiment, *J. S. Stokes*; Insulation of Observatory Domes for Protecting Telescopes and other Physical Apparatus against Unnecessary Extremes of Heat and Cold, *David Todd*; Note on the Coefficients of Diffusion of the Emanation and the Active Deposit Particles of Antinium, *J. C. McLennan*; A Quantitative Study of the Emission of Hydrogen, Argon, and Helium in the Visible Spectrum, *P. G. Nutting and Orin Tugman*.

THE PHYSICAL REVIEW
LANCASTER, PA., AND ITHACA, N. Y.
BERLIN: MAYER AND MUELLER

THE PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND THEORETICAL PHYSICS

Conducted

With the Co-operation of the

AMERICAN PHYSICAL SOCIETY

BY

EDWARD L. NICHOLS

ERNEST MERRITT, AND FREDERICK BEDELL

Two volumes of THE PHYSICAL REVIEW are published annually, these volumes beginning in July and January, respectively, and containing six numbers each. The price of subscription is three dollars a volume, or six dollars a year. Subscriptions should be sent to the publishers, THE PHYSICAL REVIEW, 41 *North Queen Street, Lancaster, Pa.*, or *Ithaca, N. Y.*; Messrs. MACMILLAN & CO., LTD., *London;* or to Messrs. MAYER AND MUELLER, *Berlin.*

Beginning with Vol. XVI THE PHYSICAL REVIEW has been conducted with the co-operation of the AMERICAN PHYSICAL SOCIETY. The proceedings of the Physical Society are published regularly in the REVIEW.

Previous to Volume V (July–December, 1897) THE PHYSICAL REVIEW was published in annual volumes, each containing six bimonthly numbers, beginning with the July–August number, 1893. These may be obtained from the publishers at the former subscription price, three dollars per volume. From 1898 to 1908 inclusive (volumes VI to XXVII) the price of the Physical Review was two dollars and fifty cents per volume, or five dollars a year. Copies may be obtained from the publishers at this price.

Correspondence relating to contributions should be addressed to the editors at *Ithaca, New York.* If an article submitted for publication in THE PHYSICAL REVIEW is to be published elsewhere, notice to that effect should be given.

The authors of original articles published in the REVIEW will receive one hundred separate copies in covers, for which no charge will be made; additional copies, when ordered in advance, may be obtained at prices depending upon the length of the article, etc. A schedule of prices may be obtained from the editors.

THE PHYSICAL REVIEW

Entered at the post-office, at Lancaster, Pa., as second class matter.

Volume XXX. *May, 1910.* *Number 5.*

THE

PHYSICAL REVIEW.

THE TERMINAL VELOCITY OF FALL OF SMALL SPHERES IN AIR.

By John Zeleny and L. W. McKeehan.

§ 1. Introduction.

THE resistance which a fluid offers to a body moving through it increases with the speed, and hence a body falling freely through a fluid attains a limiting velocity called the terminal velocity of fall, when the resistances to motion become equal to the body's resultant weight.

Stokes[1] obtained the expression $6\pi\mu aV$ for the resistance experienced by a spherical body, when all other resistances are negligible compared to that due to the viscosity of the fluid, and when no slipping is supposed to occur at the surface of separation. In this expression, μ is the coefficient of viscosity of the fluid, a the radius of the sphere, and V its velocity through the fluid. The terminal velocity of a freely falling sphere, obtained by putting the above value of the resistance equal to the resultant weight of the sphere, is

$$V = \tfrac{2}{9}\frac{ga^2(\sigma - \rho)}{\mu},$$

σ being the density of the sphere, and ρ that of the fluid. The exclusion of all but viscous resistance restricts the applicability of the formula to very minute spheres, the condition to be fulfilled being that the radius of the sphere must be small compared to $\mu/V\rho$.

In recent years Stokes' formula has been used in important meas-

[1] G. G. Stokes, Mathematical and Physical Papers, Vol. III., p. 59.

urements for determining the size of minute spheres from their rate of fall in air, and it is the object of the experiments to be described, to test the validity of the formula when air is the fluid. Such a test requires the measurement of the radius, the density, and the terminal velocity of the spheres used, the density and viscosity of air being known. The necessity of using spheres of microscopic size gives rise to the chief difficulties of the experiments. The first of these is to obtain spheres which are sufficiently small for the purpose.

§ 2. EXPERIMENTS WITH SPORES.

The microscopic spores produced by certain plants are quite spherical in some species, and so were chosen as suitable for testing Stokes' formula. The choice is limited, however, to such spores as not only approximate to a spherical shape but also are obtainable free from other material in a sufficient quantity to permit an accurate determination of density to be made. The three kinds of spores selected were those of lycopodium, lycoperdon, and polytrichum.

Lycopodium was used first because it is obtainable as a commercial powder, very free from other material. The individual spores are only roughly spherical, their shape being shown in the microphotographs[1] (Plate, Figs. 1 and 2). Fig. 1 shows that the outer cell walls are free from any hair-like projections.

All the lycoperdon spores used were obtained from one large, well-seasoned, Colorado puff-ball. The spores were gotten practically free from impurities by rubbing masses of the puff-ball on a cloth sieve. These spores are, on the whole, very spherical, and are among the smallest known. A group of them, magnified 900 diameters, is shown (Plate, Fig. 3).

The polytrichum spores (Plate, Fig. 4) were obtained from some freshly gathered plants.[2] The ferns were dried, and the spores collected by cutting open the capsules and shaking them on a sieve. The smoothness of the walls of the spores is to be noticed in the microphotograph. The spores were not perfectly seasoned when

[1] All of the microphotographs shown were kindly made for us by Prof. E. W. D. Holway.

[2] Kindly gathered for us by Prof. C. O. Rosendahl.

used, since, several months after the close of the experiments with them, changes in the shape of some were observable.

The question will now be considered with what accuracy it is possible by the aid of Stokes' formula to calculate the terminal velocity of fall of particles which deviate somewhat from the spherical shape. When such a particle is taken to be equivalent to a sphere having a diameter equal to the average diameter of the particle, the calculated terminal velocity may be in error for two reasons; the calculated volume may differ from the true volume, and the calculated resistance to motion may differ from the true

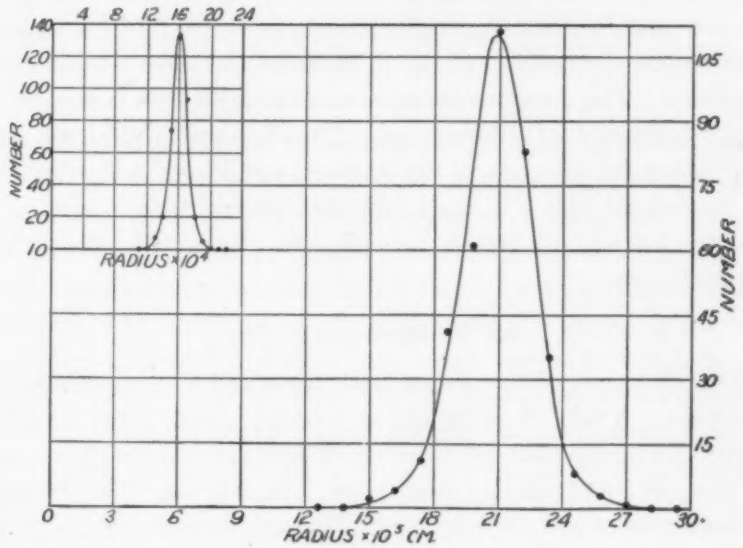


Fig. 1.

resistance. The magnitude of these errors may be learned by considering a body of some regular shape other than spherical. As an extreme case take an ellipsoid[1] whose semi-axes are 4, 5 and 6. The volume of the assumed equivalent sphere (radius 5) is four per cent. greater than the true volume. The resistance to motion in the direction of the shortest axis is equivalent to that of a sphere of radius 5.2 instead of 5, and for a motion in the direction of the longest axis the resistance is equal to that of a sphere of radius 4.8.

[1] For the equations to be used see Lamb, Hydrodynamics, p 534.

Taking into account the differences due to both the volume and the resistance to motion, the terminal velocity of the ellipsoid, calculated on the supposition that it is equivalent to a sphere having a radius the average of the three axes, is eight per cent. larger than the velocity calculated for the ellipsoid when moving in the direction of its shortest axis, and the same as that of the ellipsoid moving in the direction of its longest axis. Now the shape of the spores used is much more nearly spherical than that of the supposed ellipsoid, so that the error involved in assuming them spheres cannot be, in any case, more than a minor fraction of that found for the ellipsoid moving in the direction of its shortest axis. Since the spores are not homogeneous the axial direction in which they fall depends partly upon the position of the oil globules and other bodies within the walls. The arrangement of some of these bodies is seen in the microphotograph of polytrichum. Thus in a group of spores some may fall in the direction of the shortest, and others in the direction of the longest axis. It must be noted, however, that here, as in the development of Stokes' formula, the absence of eddy motion is presupposed.

(*a*) *Measurement of Size.*

The diameters of the spores were measured with an ocular micrometer. A field of particles was traversed, and as the cross-hair moved across the field the diameter of each particle was determined in turn, in its accidental orientation. In this way the diameters of some 300 to 400 particles were obtained for each kind of spore. With the smallest spores magnifications of over 1,000 diameters were used.

Lycopodium and lycoperdon were measured dry. The latter and polytrichum were measured when immersed in lactic acid for better definition. The results by the two methods for lycoperdon were in agreement, though it appears from the work of Buller,[1] since published, that in some cases of this kind differences in size due to mounting have been observed.

The individual spores vary considerably in size, and the curve

[1] A. H. R. Buller, Researches on Fungi, 1909. An account is also given here of the author's experiments on the velocity of fall of various spores.

in Fig. 1 shows in what proportion the different sizes occur in the case of lycoperdon. The number of particles per .000012 cm. variation in radius is plotted as the ordinate of the curve at the center of the range indicated. The curve closely resembles a probability curve. The corresponding curves for lycopodium and polytrichum are quite similar to the one for lycoperdon; that for lycopodium is shown on a reduced scale in the upper left-hand corner of the figure.

The values obtained for the average radius of the three kinds of spores are as follows:

Lycopodium	.00158 cm.
Polytrichum	.000478 cm.
Lycoperdon	.000209 cm.

(*b*) *Measurement of Density.*

The average density of the spores was obtained by determining the volume of a known mass of the substance by means of a volumenometer. The usual form of Regnault's volumenometer was at first used, but later an improved form was constructed having a baromanometer for measuring the pressures, thus making these independent of the fluctuations of the atmosphere. By using this instrument in a constant temperature room, results in which much more confidence could be placed were obtained with less labor.[1] The determinations with the two instruments, both of which were used for two of the substances, were, however, in good agreement. The volume of the substance measured varied from 3 c.c. to 10 c.c., and the following were the densities obtained:

Lycopodium	1.175
Lycoperdon	1.44
Polytrichum	1.53

(*c*) *Terminal Velocity of Lycopodium.*

Since only the average densities and diameters of the spores were determined it became necessary to obtain their average terminal velocities. The apparatus used for lycopodium is shown in Fig. 2.

The lycopodium was started on its fall in the box *A*, and after passing through the tube *B*, fell upon a moving strip of black paper *CC*. This strip of paper, held tight by the weights *WW*, was moved

[1] J. Zeleny and L. W. McKeehan, PHYS. REV., XXX., p. 189, 1910.

uniformly by the chronograph drum D. A piece of white paper, pasted at one side of the black paper strip, received the time record from the two pens PP'. One of these marked the seconds from the clock, and the other recorded the moment when the powder was released. The portion of the strip on which the powder fell was surrounded by the metal case EE to shield it from air currents. Two metal rods FF' held the paper up against the lower end of the

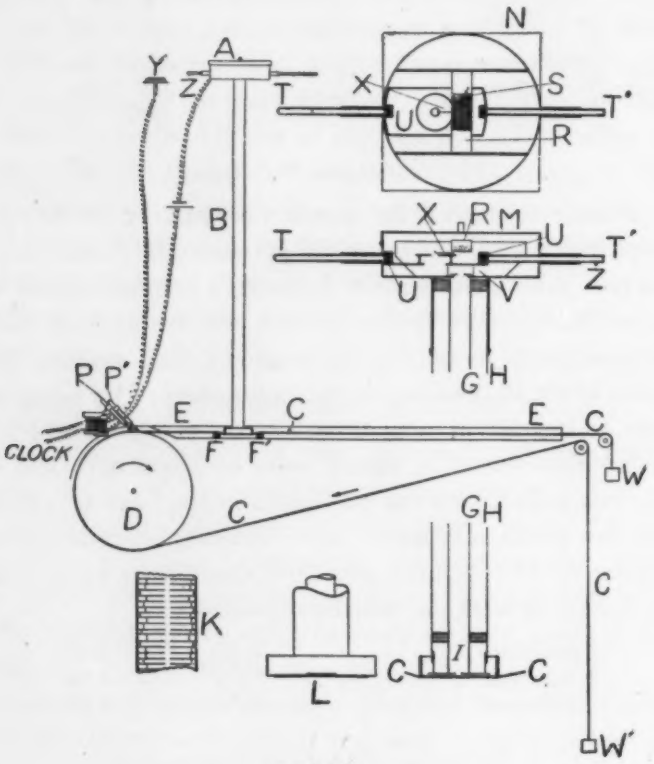


Fig. 2.

fall tube. This portion is shown enlarged at the lower part of the figure. The fall tube consisted of the two concentric tubes GH, the outer of which protected the inner from temperature changes. The length of both tubes was usually about 71 cm.; the diameter of the outer one was 3.5 cm., and that of the inner was varied. The lower part of the latter was closed by a cap having a small

hole *I*, this permitting only such particles to pass through onto the paper *CC* as had travelled through the central portion of the tube, and so had suffered no retardation from its sides. The adjacent drawing, representing the tube turned 90° on its axis, shows the shallow opening *L*, through which the particles that had fallen onto the paper could pass without being rubbed off by the metal parts above them. The details of the box *A* from which the powder was started are shown in elevation at *M* and in plan at *N*, the position shown being 180° from that at *A*. The lycopodium powder was placed above the center of the fall tube in the metal tray *R*, the bottom of which was made of a tightly stretched silk cloth *S*. The rods *TT'*, which were movable in the tubes shown, carried the flat slider *U*. This slider just grazed the top of the short cylinder *V*, which had a perforated diaphragm in it. There was a hole in the center of the slider above which was a short cylinder, closed by a lid with a small hole in its center. Projecting over this hole in the lid and fastened to the edge of the lid itself, was the flexible pin-point *X*. When the slider was pushed to the right (in the enlarged figure) the pin would scratch lightly on the bottom of the cloth, allowing some of the lycopodium particles to shift into the fall tube through the small hole in the slider, when the latter was in its central position. The point *Z* at this instant made an electric contact with *Y*, actuating the pen *P*. The motion of the slider past its central position again closed the top of the fall tube. A rubber band drawn around the edge of the lid of the box *A*, and oil in the slide tubes, made the upper portion of the apparatus air-tight and prevented the passage of a convection current up or down the tube.

The method of conducting an experiment was as follows. After the paper strip *CC* was set into motion by the drum *D*, the slider was pushed by the rod *T*, freeing some of the particles. The motion of the paper was allowed to continue until previous experience had taught that all of the particles had reached the bottom. The drum was then stopped, the part of the paper carrying the record was cut off from the rest, placed on a smooth surface, and carefully covered with a strip of glass to prevent any subsequent displacement of the particles. These particles could easily be seen against the black paper with a low power microscope, and the time taken

for each particle to fall was determined from the time record at the side. The average time taken to fall the known length of tube gave the average terminal velocity. The number of particles obtained on the paper at any trial depended upon how hard the point X scratched the cloth. Only light scratches were used, as otherwise many thousands of particles fell to the paper. In the preliminary experiments several other methods of starting the particles were tried, but the one described above was finally adopted as the best.

The fall tube was modified in a number of ways. Inner tubes of different diameters were used, ranging from 3 mm. to 25 mm. Again, the outer tube was filled with centrally perforated paper discs, separated by paper rings as indicated by the section K in Fig. 2. This arrangement was designed to prevent air circulation in the tube and yet diminish any effect which might exist in the small fall tubes due to the proximity of the wall of the tube. The height of fall of the particles was varied by employing fall tubes of lengths 71 cm. and 211 cm. As the results show, none of these variations had any marked effect on the value obtained for the terminal velocity of the particles.

The particles of lycopodium adhere to each other quite readily, so that in addition to the single particles a number of groups of particles would start to fall at the same time. The largest of these would reach the bottom first and then would come the smaller groups, ending in threes and twos, and finally single particles. It was attempted to avoid the liberation of large groups by scratching the cloth very lightly.

Fig. 3 shows an example of the distribution of the particles on the paper strip in respect to time of fall, the number falling in any second being represented by the ordinate at its end. Curve A shows the distribution of the single particles, curve B, the doubles, and curve C, the triples. The average velocity of the singles obtained from this set of particles is 1.76 cm./sec., that of the doubles is 2.40 cm./sec. The doubles thus fall 1.37 times as fast as the singles. An unsymmetrical form of distribution curve for the time of fall is to be expected, although the size distribution curve is symmetrical (see Fig. 1), because the time of fall of a particle should

vary inversely as the square of its radius. Thus three particles whose sizes are in the ratio of $2/3 : 1 : 4/3$ should have times of fall in the ratio $9/4 : 1 : 9/16$, a series which is no longer arithmetical. The extreme times of fall of the particles represented by the curve *A* of Fig. 3 are in about the ratio to be expected from the distribution of sizes (Fig. 1), but the fact that the number of particles to the right of the maximum in Fig. 3 is greater than that to the left indicates that the smaller particles are, on the whole, the denser ones. A summary of the results obtained for the terminal velocity of fall of lycopodium is given in Table I (p. 544). The results obtained under the various conditions are seen to be in satisfactory agreement.

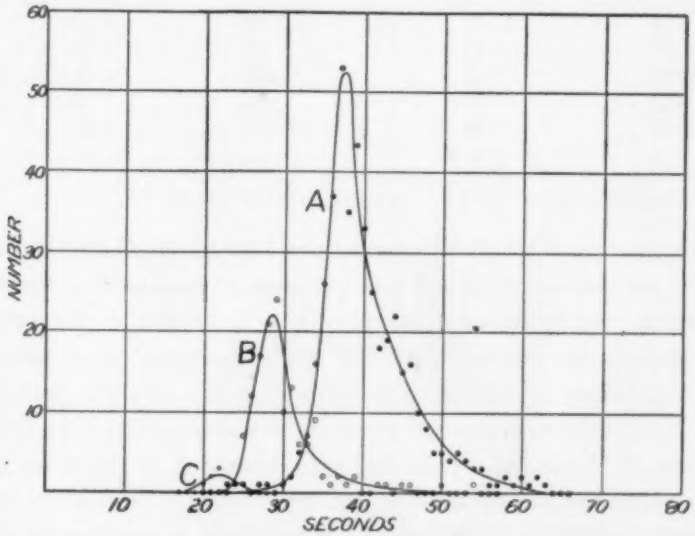


Fig. 3. Temperature 21°.1 C. Length of tube 71.5 cm. Average time, single particles 40.7 sec. Terminal velocity 1.76 cm./sec. Average time, doubles 29.8 sec. Terminal velocity 2.40 cm./sec.

(*d*) *Terminal Velocity of Lycoperdon.*

The minuteness of the lycoperdon particles made it impossible to employ for them the apparatus which was used to determine the velocity of fall of lycopodium, as it was almost impossible to find the particles after they had fallen, with the high power microscope required to see them. A new apparatus was constructed, which is shown in Fig. 4 (p. 545).

TABLE I.

Lycopodium.

Length of Tube.	Diameter of Tube.	Number of Particles.	Terminal Velocity of Fall.
71.5 cm.	.32 cm.[1]	26	1.72 cm./sec.
71.5	.32[1]	18	1.77
71.5	.32	162	1.70
71 5	.32	133	1.70
71.5	.64	30	1.71
71.5	.64	170	1.73
71.5	1.27[2]	354	1.76
71.5	1.27[2]	431	1.76
71.5	1.27[2]	327	1.78
71.5	1.27[2]	267	1.74
71.5	1.27[2]	359	1.80
71.5	1.27	66	1.76
71.5	1.27	60	1.74
210.9	1.27	462	1.82
71.5	2.54	34	1.74
71.5	2.54	138	1.80
71.5	2.54	52	1.78

Average temperature 22°.4 C. Average terminal velocity 1.76 cm./sec.

The upper portion *A*, which contained the arrangement for starting the particles on their fall was the same as that used in the other apparatus, and is shown in detail in Fig. 2. From *A* the particles fell through the innermost of the four concentric brass tubes *B*, the total height of fall being usually 31.2 cm. At the bottom of the fall tube the particles fell upon the revolving plate glass disc *C*, 12.7 cm. in diameter. This disc was cemented to the brass plate *D*, to which the axis of rotation was attached. The grooved wheel *E* was fastened to the same axis and was kept in rotation by a belt from a chronograph motor, through a speed reducer. The time of one revolution was about twenty minutes. A paper disc was attached to the top of the wheel *E*, and a stationary pen (not shown), actuated by a clock, marked second intervals upon the paper. The plate glass disc was completely enclosed in a metal box *F*, so that the whole space inside the apparatus was practically air-tight. Opposite the fall tube *B*, the box *F* had an opening for observing the particles with the microscope *M*. A section of this part is

[1] Size of apertures in paper discs .35 cm. apart.
[2] Size of apertures in paper discs .64 cm. apart.

shown enlarged at the lower right-hand corner of the figure. The opening in the cover of the box was closed by a thin microscope cover-glass *G*. The upper surface of the glass disc *C* revolved very close to this cover-glass, enabling a high power objective *M* to be used. Opposite the upper opening there was a corresponding window in the lower wall of *F*, which was closed by the glass plate *K*.

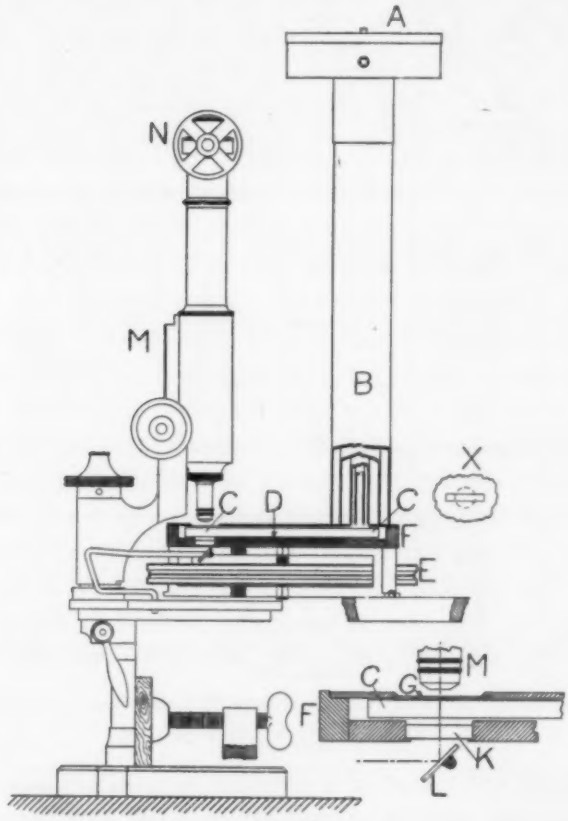


Fig. 4.

Light was reflected up from the mirror *L* to illuminate the field of the microscope. As the plate was slowly revolved the particles on its surface came into the clear microscope field where they could be seen very clearly, and their diameter could be measured, if desired, with the micrometer eyepiece *N*. The motion of the glass disc was so true that as it revolved its surface could readily be kept

in focus. An additional window, which is not shown, closed by a removable cover, was provided in the top of the box F for removing the particles from the glass disc at the end of any experiment. The opening at the lower end of the fall tube was closed with a diaphragm having a slot 3 mm. wide (shown at X). This not only permitted a more exact determination of the time of fall, but also prevented particles which fell along the sides of the fall tube from alighting among those which had fallen through the center of the tube.

Inasmuch as the time required for some of the smallest particles to fall the distance of 31.2 cm. was 15 minutes, great precautions were taken to guard against convection currents in the fall tube.

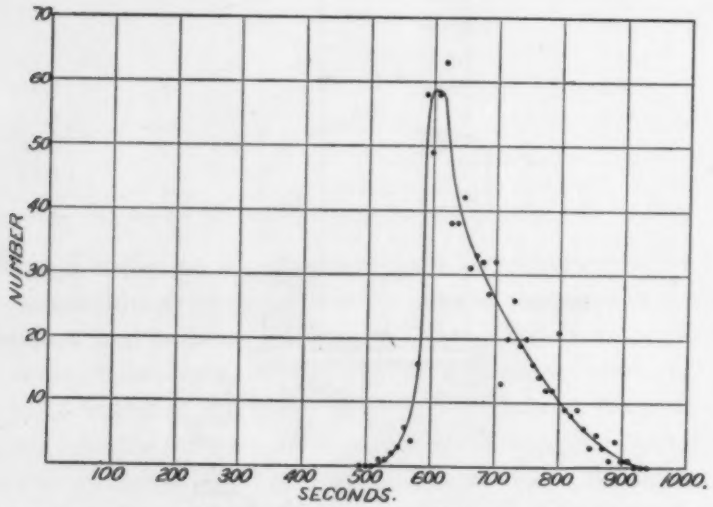


Fig. 5. Temperature 20°.8 C. Length of tube 31.2 cm. Average time of fall 678 sec. Terminal velocity .0460 cm./sec.

The apparatus was placed in a constant temperature room, and in no case were the particles started on their fall until several hours after the adjustments had been made. The operator entered the room only long enough to begin the experiment. After starting the chronograph motor, disconnected from the running gear, the particles were released, and at the same instant the running gear was closed, setting the disc into slow rotation. The time-recording pen thus began to trace its record at the instant the particles started to fall.

After the particles had fallen, the disc was turned to a determined position which placed that part of it under the microscope which in the initial part of the experiment had been under the center of the fall tube. A stationary index point was then placed opposite the beginning of the time record upon the paper. When the disc was now turned until a particle appeared in the center of the microscope field, the index showed the time it had taken that particle to fall. Starting with the initial position of the disc, this was slowly turned and the time taken for the different particles found. Then the microscope was moved radially the width of one field and the process repeated until the time of fall of all the particles on the disc had been determined. An illustration of the nature of the results obtained with lycoperdon is shown in Fig. 5.

The time interval was divided into ten-second spaces, and the number of particles found in each of these intervals is plotted opposite the end of the interval. The curve representing the distribution on the disc is similar to that obtained for lycopodium. There is the same indication here, too, that the smaller particles are, on the whole, denser than the larger ones. The height of fall was 31.2 cm. in all but two experiments in which it was changed to 10.4 cm. without, however, altering the value of the velocity obtained. In some of the experiments a tube of radium was placed in the starting box to discharge any electrification which the particles might get from the cloth in passing through it, but this too had no effect upon the value obtained for the terminal velocity. A summary of the results obtained for lycoperdon is given in Table II.

TABLE II.

Lycoperdon.

Diameter of Tube.	Number of Particles.	Terminal Velocity.
1.27	280	.0463 cm./sec.
1.27	757	.0462
.64	713	.0473
.64	104	.0480
.64	75	.0470
.64	75	.0475
.64	216	.0457
.64	652	.0460
.64	292	.0465
.64	186	.0468
Length of tube 31.2 cm.	Average temperature 20°.5 C.	Average terminal velocity .0467 cm./sec.

The velocity of lycoperdon is so small that it was thought possible to determine it by observation of the fall across the field of a microscope. A square glass cell 1 mm. in cross-section and 8 cm. deep was made and set up in front of a horizontal microscope. A piece of the puff-ball was placed in the opening at the top so that tapping the cell started a shower of spores down the narrow channel. The lower portion of the space was quite free from air currents, as the individual particles descended in straight lines. No Brownian movements could be observed, nor was any rotation of the single particles noticed, though there was rotation of some of the doubles. The time required for the particles to move a distance of 2 mm. was taken with a stop-watch, and an average velocity of .05 cm./sec. was obtained. This value agrees roughly with the result of the more accurate method. The method of direct observation is, however, unsatisfactory for two reasons. The measurement of the time for the particles moving rapidly across the field is difficult, and it is impossible by timing individual particles chosen at random out of a group which started to fall at the same time, to obtain the average velocity of the group, the fraction of the total number that have any observed velocity being unknown.

(e) *Terminal Velocity of Polytrichum.*

The experiments with polytrichum were carried out in the same way as those with lycoperdon. The curves showing the distribution of the particles as to time of fall are similar to those for lycoperdon, except that the maxima are not as sharply marked. A summary of the results obtained with a fall tube .64 cm. in diameter and a height of fall of 31.2 cm., is given in Table III.

TABLE III.

Polytrichum.

Number of Particles.	Terminal Velocity.
35	.221 cm./sec.
15	.229
1551	.238
1674	.218
2337	.238
832	.250
2653	.226
668	.215

Average temperature 25°.3 C. Average terminal velocity .229 cm./sec.

(f) *Comparison with Theory.*

In Table IV., a summary of the various results for spores is given, and the observed values of the terminal velocities of fall are compared with those calculated by the formula

$$V = \tfrac{2}{9}\frac{ga^2(\sigma - \rho)}{\mu}.$$

The value of μ used in the calculations ($\mu = .000191$ at 18° C.) is that recently obtained by Zemplèn,[1] which is somewhat larger than that heretofore accepted.

TABLE IV.

Comparison of Observed and Calculated Velocities for Spores.

Name.	Radius.	Density.	$\frac{a V\rho}{\mu}$	Average Temp.	Average Vel. Observed.	Velocity Calculated.	Ratio $\frac{V \text{ calc.}}{V \text{ obs.}}$
Lycopodium	.00158	1.175	.02	22°.4	1.76	3.40	1.93
Polytrichum	.000478	1.53	.0007	25°.3	.229	.396	1.73
Lycoperdon	.000209	1.44	.00007	20°.5	.0467	.0718	1.52

The ratio of a to $\mu/V\rho$ is given in the fourth column to show that the spheres fulfill the size criterion for the applicability of the formula, which is that "a" should be small compared to $\mu/V\rho$.[2] It is seen that the calculated velocity is in each case much larger than the value observed, the difference being greater for the larger particles.

§ 3. EXPERIMENTS WITH ARTIFICIAL SPHERES.

(a) *Black Wax.*

Because of the discrepancies found in the case of spores between the observed and calculated values of the terminal velocity, attention was next given to the making of artificial spheres. It was found that paraffin wax when melted could be broken up by an atomizer into minute spherical drops which retained their shape on solidifying, although the attendant contraction left the surface of the spheres somewhat rough. A hard, brittle, black wax, of unknown composition, was next utilized and proved to possess admirable

[1] Gy. Zemplèn, Annalen der Physik, Vol. 29, p. 869, 1909.

[2] Rayleigh, Phil. Mag., V., 36, p. 365, 1893.

qualities for the purpose in hand. The perfect spheres obtained (Plate, Fig. 5) have a very smooth surface, and could be made in all sizes desired. The method of making and collecting the spheres was as follows. A piece of the wax was melted in a beaker (A, Fig. 6), and heated considerably above the melting point (to about 150° C.) in a sand-bath B. The atomizer C was made of two glass tubes, the size and adjustment of whose openings had much to do with the size of the spheres obtained. Unless the temperature of the vertical tube was maintained sufficiently high by an occasional application of a flame, the blast of air would cool the wax so that it would be drawn out into long fibers, or perhaps close the end of the tube completely. The atomized wax was blown by the blast itself into the end of a long inclined glass tube E about 3 cm.

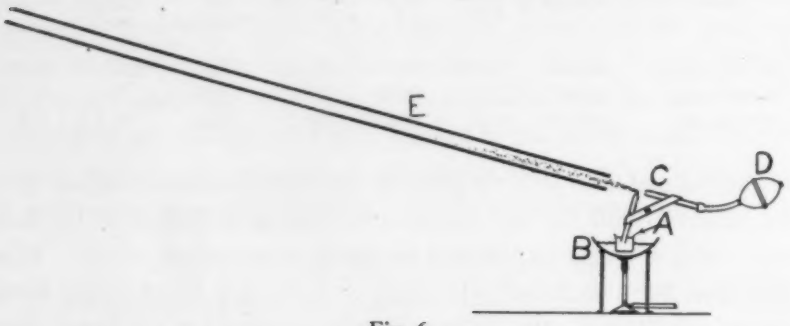


Fig. 6.

in diameter and 150 cm. long. As the air passed up this tube many of the suspended spheres settled on the wall, while the very finest ones issued from the upper end as a thin smoke. Some of these could be caught in a loose plug of cotton placed in the upper end of the tube, but nearly all of them were too small for accurate measurement. To collect the particles, the glass tube E was held vertically over a paper and tapped. An enormous number of perfect spheres of a large range of diameters was thus obtained.

The density of the wax was found, by weighing in water, to be 1.058 at 20° C.

The apparatus of Fig. 4 was used for determining the terminal velocity of fall, a supply of the collected spheres being placed in the trough in the starting box (see R, Fig. 2). The only difference

between the conduct of the experiment with the new material and with the spores was that now it was necessary to measure the diameter of each particle when found on the rotating disc, in addition to getting its time of fall. The particles are somewhat transparent,

TABLE V.

Black Wax.

Radius of Sphere.	Time of Fall Observed.	Time of Fall Calculated.	Radius of Sphere.	Time of Fall Observed.	Time of Fall Calculated.
cm.	sec.	sec.	cm.	sec.	sec.
.00366	3.4±6	1.9	.00113	21.1	20.2
.00218	6.3	5.4	.00105	22.5	23.4
.00217	6.6	5.5	.00087	32.6	34.0
.00211	6.2	5.8	.00075	47.3	45.8
.00205	6.9	6.2	.00070	54.4	52.6
.00197	6.4	6.7	.00069	54.9	54.0
.00195	7.9	6.8	.00067	61.9	57.4
.00194	7.1	6.9	.00067	62.7	57.4
.00191	8.5	7.1	.00066	57.1	59.1
.00181	10.1	7.9	.00064	60.6	62.9
.00178	8.0	8.2	.00059	67.8	74.0
.00172	8.2	8.7	.00058	75.8	76.6
.00169	8.8	9.0	.00054	82.5	88.4
.00167	9.9	9.3	.00050	97.3	103.2
.00165	10.6	9.5	.00049	105	107
.00165	11.1	9.5	.00049	113	107
.00156	11.4	10.6	.00048	116	112
.00151	12.2	11.3	.00046	120	122
.00147	11.4	11.9	.00045	118	127
.00146	12.4	12.1	.00044	152	133
.00145	13.4	12.2	.00042	135	146
.00142	13.3	12.8	.00041	144	153
.00130	15.6	15.2	.00039	166	169
.00130	15.7	15.2	.00038	184	178
.00124	16.8	16.8	.00036	191	199
.00121	17.4	17.6	.00035	166	211
.00119	13.0	18.2	.00035	227	211
.00119	16.6	18.2	.00035	231	211
.00117	13.6	18.8			

Average temperature 20°.0 C. Length of fall 31.2 cm. Diameter of tube .64 cm.

and the surface does not reflect so well but that a sharply outlined image is obtained, permitting accurate readings to be taken. Some of the particles were found to contain one or more air bubbles, and all particles where these constituted an appreciable part of the

volume were discarded, together with those to which any dust was found adhering. Owing to the width of the slit at the bottom of the fall tube (see X, Fig. 4) the error in the time of fall of any one particle may be as much as $\pm .6$ sec. Table V. gives a summary of the results obtained for all of the particles in four different experiments.

The values given in the third column were calculated by the aid of Stokes' formula, using .000192 for the value of μ at 20° C. The limiting size to which the formula may be applied is found by putting

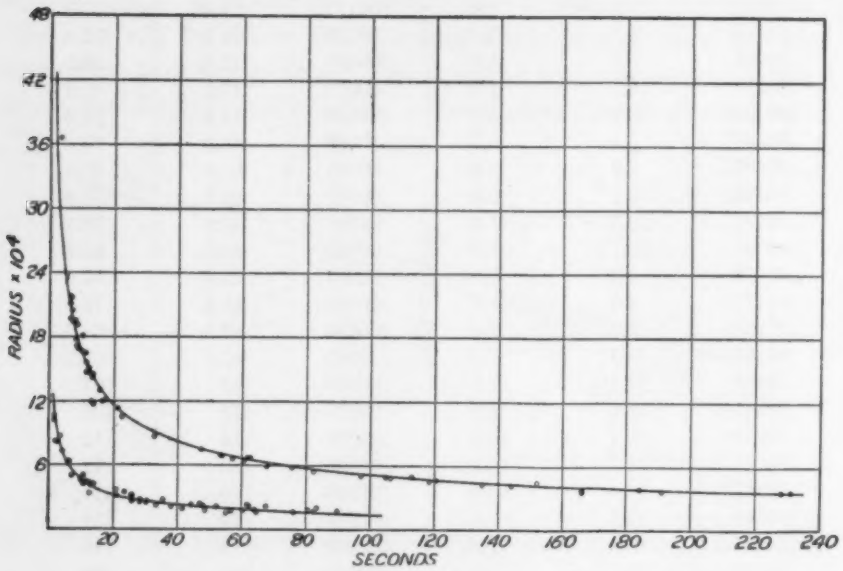


Fig. 7.

$aV\rho/\mu = .1$[1] (calling .1 a small quantity). This corresponds to a radius of approximately .0022 cm. All of the particles except the first one are within this limit. The results given in Table V. are shown graphically in the upper curve of Fig. 7, the various circles representing the individual observations, and the continuous curve, the theoretical values. The deviation of a few of the individuals from the theoretical curve is large, but on the average the deviation is less than one half of one per cent. The results covering such a large range of sizes show quite conclusively the applicability of Stokes' formula to these spheres.

[1] See reference to Rayleigh above.

Some of the black wax particles are so large that it is necessary to calculate the error that may arise because of the time taken for the particle to attain the terminal velocity. The equation of motion for the falling sphere is

$$\frac{dV}{dt} + kV = g,$$

where

$$k = \frac{6\pi\mu a}{\frac{4}{3}\pi a^3(\sigma - \rho)} = \frac{9\mu}{2a^2(\sigma - \rho)},$$

whence

$$V = \frac{g}{k}(1 - \varepsilon^{-kt}),$$

where g/k is the terminal velocity. We can find how long a time elapses before the velocity of the sphere is within one per cent. of the terminal velocity, by finding the value of t for which ε^{-kt} is equal to .01.

For the largest sphere to which Stokes' formula is applicable

$$a = .0022 \text{ cm.},$$

and therefore

$$k = 165,$$

so that $t = .03$ sec. This is less than .6 per cent. of the total time of fall, hence the error introduced by assuming that the terminal velocity is instantly attained is still less than this, and may safely be neglected.

(*b*) *Mercury.*

Some experiments were next performed with minute mercury drops which were also made by means of an atomizer. The time of fall and the diameter of the drops were determined with the same apparatus in the same way as for the black wax. In the measurement of diameter, however, a serious difficulty arose because of inability to get well-defined images of the drops, owing to the high reflecting power of the mercury. The best system of illumination under the restricted conditions was found to be a side illumination together with a faint illumination from below. Nevertheless, the errors of the measurements were quite large, especially so, because in order that Stokes' formula may be applied to such a dense substance extremely minute drops have to be used.

The results obtained for the mercury drops in several experiments, together with theoretical values, are given in Table VI.

TABLE VI.

Mercury.

Radius of Sphere.	Time of Fall Observed.	Time of Fall Calculated.	Radius of Sphere.	Time of Fall Observed.	Time of Fall Calculated.
cm.	sec.	sec.	cm.	sec.	sec.
.001075	1.6±6	1.8	.000274	25.8	27.0
.001030	1.6	1.9	.000273	28.3	27.2
.000880	3.2	2.6	.000267	25.8	28.5
.000830	1.9	2.9	.000267	30.3	28.5
.000826	2.5	3.0	.000254	28.3	31.5
.000736	4.0	3.7	.000252	30.2	32.0
.000640	5.7	4.9	.000239	47.0	35.5
.000640	4.7	4.9	.000238	47.1	35.8
.000508	10.6	7.9	.000234	61.8	37.1
.000506	7.0	7.9	.000233	44.4	37.4
.000465	9.8	9.4	.000225	33.4	40.1
.000449	11.3	10.0	.000223	62.3	40.8
.000437	12.4	10.6	.000222	48.1	41.2
.000428	10.5	11.1	.000216	38.0	43.5
.000428	10.8	11.1	.000215	52.1	43.9
.000422	13.8	11.4	.000205	83.6	48.3
.000413	13.0	11.9	.000194	41.5	53.9
.000370	20.8	14.8	.000188	80.7	57.5
.000349	23.3	16.6	.000186	64.1	58.6
.000338	12.5	17.7	.000185	67.6	59.4
.000324	25.6	19.3	.000181	48.7	62.0
.000324	25.6	19.3	.000179	58.3	63.4
.000322	25.5	19.6	.000177	56.8	64.8
.000318	20.9	20.1	.000174	89.9	67.0
.000314	25.8	20.6	.000170	82.3	70.4
.000314	21.0	20.6	.000165	76.3	74.7
.000309	20.7	21.3	.000164	64.7	75.6
.000277	35.4	26.5	.000159	55.2	80.4
.000275	28.3	26.8			

Average temperature 20°.3 C. Length of fall 31.2 cm. Diameter of tube .64 cm.

These observations are represented graphically in the lower curve of Fig. 7, where the continuous line again shows the theoretical values. It is seen that the individual observations show very large deviations, but nevertheless the results in general cluster about the line representing the theoretical values. For a mercury drop of

radius equal to .001 cm. the ratio $aV\rho/\mu = .107$, and hence all drops smaller than this are within the range of applicability of Stokes' formula. The time required for a drop of the size stated to attain a velocity within 1 per cent. of its terminal velocity is .08 sec., which is 4 per cent. of the total time of fall in the apparatus used.

TABLE VII.

Paraffin.

Radius of Sphere.	Time of Fall Observed.	Time of Fall Calculated.	Radius of Sphere.	Time of Fall Observed.	Time of Fall Calculated.
cm.	sec.	sec.	cm.	sec.	sec.
.00415	1.5±6	1.8	.00214	6.7	6.6
.00372	3.2	2.2	.00206	7.3	7.2
.00369	2.5	2.2	.00202	8.55	7.4
.00354	3.4	2.4	.00200	7.43	7.6
.00343	3.75	2.6	.00200	7.4	7.6
.00328	2.65	2.8	.00180	8.7	9.4
.00327	3.8	2.8	.00179	9.05	9.5
.00324	4.1	2.9	.001725	10.95	10.2
.00317	3.5	3.0	.001645	10.05	11.2
.00313	3.3	3.1	.00163	10.6	11.4
.00309	3.2	3.2	.00161	11.5	11.7
.00307	4.15	3.2	.00157	12.5	12.3
.00302	3.9	3.3	.001554	12.1	12.5
.00300	3.0	3.4	.001535	11.3	12.9
.002785	3.8	3.9	.00152	14.2	13.1
.00278	3.85	3.9	.00142	15.2	15.0
.00278	4.3	3.9	.00140	15.6	15.5
.00274	5.3	4.0	.00135	18.8	16.6
.00272	5.3	4.1	.00123	18.1	20.0
.00267	4.75	4.3	.00122	22.5	20.4
.00260	3.75	4.5	.00117	21.3	22.2
.00256	4.9	4.6	.00116	20.7	22.5
.00256	1.8	4.6	.00116	20.2	22.5
.00255	4.8	4.7	.001145	24.6	23.2
.00247	5.3	5.0	.00109	27.3	25.5
.00247	5.9	5.0	.00106	25.7	27.0
.00244	6.7	5.1	.001035	29.6	28.3
.00242	5.1	5.2	.000965	30.4	32.5
.00240	5.85	5.3	.000862	45.7	40.7
.00230	5.25	5.7	.00085	38.6	41.9
.00223	6.6	6.1	.000665	77.7	68.4
.00221	6.3	6.2	.000579	92.1	90.4
.00220	6.5	6.3	.000527	106.0	109.0
.00216	7.3	6.5	.000500	123.2	121.4

Average temperature 20°.3 C. Length of fall 31.2 cm. Diameter of tube .64 cm.

(c) *Paraffin.*

The spherical particles made with the atomizer from paraffin wax differ from those of black wax and mercury in not having perfectly smooth walls. The contraction of the paraffin on cooling leaves the walls quite rough and even more or less jagged in some cases (Plate, Fig. 6). The terminal velocity of a number of these spheres was determined to see if this roughness has any effect upon their rate of fall. The density of the paraffin was obtained in bulk by weighing in water and found to be .906. In determining their time

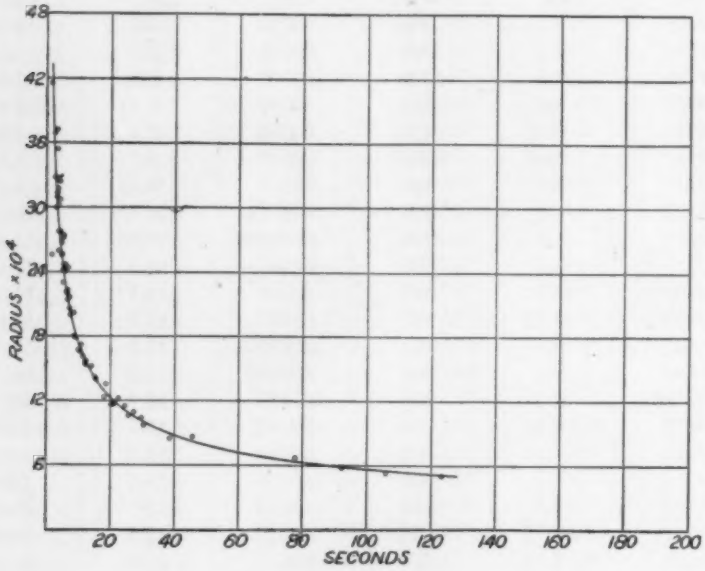


Fig. 8.

of fall, the spheres were not placed inside the trough R (Fig. 2), because of their adhesive property. Instead, the stream of atomized particles was allowed to impinge directly upon the under side of the trough, to which they adhered. The starting pin-point simply dislodged them from this position. The results are given in Table VII., together with values calculated by the formula.

The limit of size for the applicability of the formula is found to be .0024 cm., so that many of the larger particles in this table are outside the limit. For these the actual resistance to motion

should be appreciably greater than that taken into account in calculation, and the results show that this is the case. The average deviation from the theoretical values for the particles within the limit is less than 1 per cent. The results are represented graphically in Fig. 8, where again the continuous curve gives the values by Stokes' formula.

It appears from a comparison of these results with those for the black wax spheres, that roughness of surface of the order of that found in these paraffin droplets is without effect upon the terminal velocity of fall, and hence that it does not start eddy motion for the velocities used.

§ 4. Discussion of Results.

Since the terminal velocity of the artificial spheres agrees so well with the theoretical formula, the discrepancy observed with spores calls for some special explanation. How large this discrepancy is can best be seen by referring to the time of fall distribution curves for lycopodium (Fig. 3) and lycoperdon (Fig. 5). The theoretical time of fall of the average lycopodium particle is 21 sec., whereas the curve shows that not even one of the fastest of the particles had fallen within that time. The theoretical time of fall of the average lycoperdon particle is 434 sec., whereas the fastest one out of the whole group took 520 sec. to fall, the others varying in order up to 910 sec. The agreement among the separate determinations of the terminal velocity for the same substance under various conditions precludes the possibility of any large error entering into the results for this quantity. As further evidence one of the experiments with mercury drops will be cited, in which a few black wax particles and a considerable number of lycopodium particles fell in the tube at the same time. The times of fall of the wax spheres agreed closely with the calculated times, and are, indeed, included under the results given for black wax. The average velocity of the 44 lycopodium particles was 1.73 cm./sec., which agrees well with the results obtained previously with the other apparatus. Of the three kinds of particles falling here under exactly identical conditions the wax and mercury spheres had velocities agreeing with Stokes' formula, while the lycopodium fell with barely more than half of the theoretical velocity. The discrepancy in the case of the latter

cannot therefore be due to convection currents, or other errors in the determination of the velocity of fall.

It does not seem possible to attribute the difference between theory and experiment to errors in the measurements of the diameters of the spores, since that would require this measurement to be about 40 per cent. too large in the case of lycopodium, and 24 per cent. too large for lycoperdon. In these two cases, at least, the spores were assuredly of the same size in the fall experiments as when measured. They were perfectly seasoned and hence undergoing no change, and they were measured dry just as they were used in the velocity determinations. Different magnifying powers were used which required separate calibration of the ocular micrometer.

In the density determinations, the question arises whether the volume obtained with the aid of the volumenometer may not be in error, because of gases which might be liberated from the surface or interior of the spores when the pressure is reduced. An experiment was carried out with glass, the surface of which is known to condense gases. The density of a sample was obtained first by weighing in water, next by means of the volumenometer when in large pieces, and finally by the latter method after it had been powdered as finely as possible in a mortar. The three results were in good agreement. Still, the behavior of spores in this respect may differ from that of glass. A possible substitute method of getting the density of the spores is by finding the density of a solution in which they will just sink. This method is obviously open to the objection that osmosis produces changes in the spores when put into the liquid, so that the density obtained for the wet spores is not their density when dry. Nevertheless, some experiments were carried out by observing the motion of individual spores under a microscope when suspended in a solution, and also by observing them in bulk in test-tubes. The unreliability of the method was at once shown by the fact that the spores behaved differently in different kinds of solutions of the same density. Thus lycoperdon sank in a saturated water solution of NaCl whose density was 1.201, while it rose in a solution of $CaCl_2$ of density 1.200, also sinking, however, in a solution of the latter substance whose density was 1.163. The density of lycopodium determined by this method was

found to be 1.09 ± 1 when a calcium chloride solution was used, and 1.13 ± 2 when a sodium chloride solution was used. With the volumenometer it was found to be 1.175, and to explain the discrepancy between the observed and calculated values of the terminal velocity the density would have to be but .61.

The effect of the shape of the spores alone remains to be considered. Evidence has been given already (§2) showing that, in the calculations, a nearly spherical particle may be represented without a large error by a sphere having a diameter equal to the average diameter of the particle. There is one respect, however, in which this equivalence may not hold. This is in the tendency to the production of eddy motion. The conditions for a stable motion are not well understood, but in the case of the sphere the perfect symmetry of the stream lines may well represent the greatest stability in the motion. The symmetry of these lines in the rear of the moving body would not be influenced appreciably by small roughnesses on the surface. Dissymmetry in the stream lines, caused by an imperfectly spherical body, must be accompanied by forces in the fluid at right angles to the direction of motion, and these may suffice to cause eddies in the rear of the body for velocities much smaller than are necessary to produce them in the case of the sphere. Some explanation of this character must at present be regarded as the most probable one, none other appearing adequate to explain the large difference between the observed and calculated terminal velocities of the spores used.

§ 5. Summary.

The average size, density, and terminal velocity of fall in air of the approximately spherical spores of lycopodium, lycoperdon and polytrichum were obtained experimentally. The observed values of the terminal velocity were much smaller than the corresponding values calculated by Stokes' formula.

Similar experiments performed with artificial spheres made from a black wax, from mercury, and from paraffin gave results for the terminal velocity, over a large range of sizes, which were in good agreement with the theoretical formula, proving its applicability under the conditions used.

No satisfactory explanation for the discrepancies in the case of spores is apparent. It may be that turbulent motion of the fluid is started more readily by a nearly spherical body than by one which is perfectly spherical.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA, December 2, 1909.

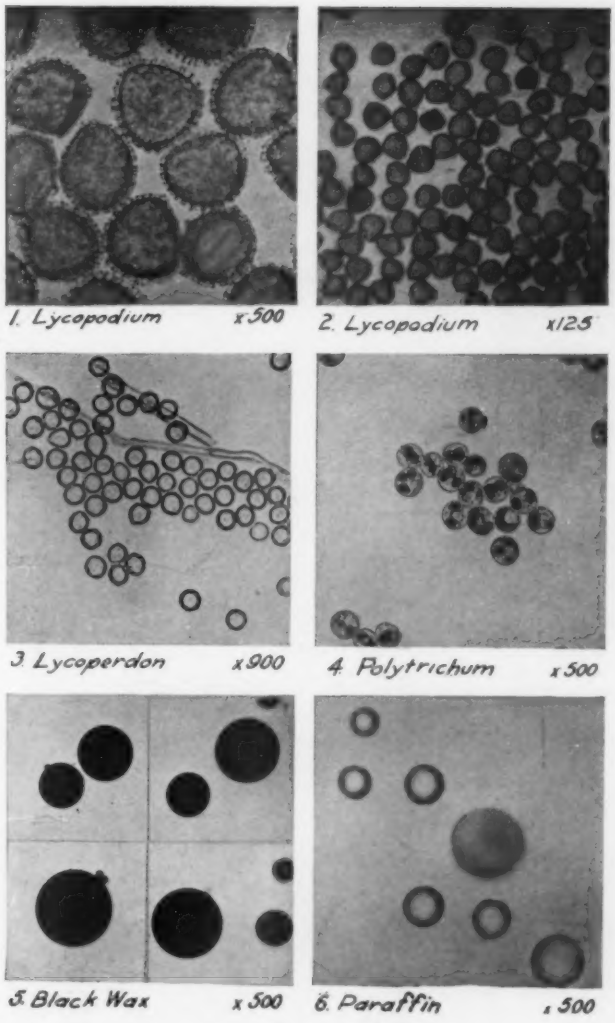


PLATE I. ZELENY AND MCKEEHAN.

A STUDY OF THE MULTIPLE REFLECTIONS OF SHORT ELECTRIC WAVES BETWEEN TWO OR MORE REFLECTING SURFACES.

By L. E. Woodman and H. W. Webb.

1. Introduction.

THE phenomenon of "multiple reflections" between two parallel reflecting surfaces is well known, especially in its application to the optics of thin films, but its application to electric waves has been investigated only within the last few years. Several investigators[1] have worked with resonators on glass plates or with two or more screens of resonators placed parallel to each other and perpendicular to the direction of propagation and have evidently not considered the effect upon their results due to the multiple reflections between the various reflecting surfaces. On the other hand Blake and Fountain[2] while working with resonators pasted on glass plates encountered the peculiar phenomenon of a larger per cent. of incident energy transmitted and a smaller per cent. reflected when the resonators were of certain lengths than was transmitted or reflected by the bare glass. Blake and Fountain attributed the phenomenon to a change of refractive index due to change of responsiveness of the medium as required by the dispersion theory. Cartmel,[3] on the other hand, gave a different explanation based on the consideration of multiple reflections between the two surfaces of the glass plate. At bottom the two explanations are but different aspects of the same fundamental phenomenon, *i. e.*, phase change and change of velocity due to the varying response of the changed character of the medium. Blair,[4] following Stokes and Fresnel, derived the formulæ for the change of phase and also

[1] Garbasso and Aschkinass, Ann. der Physik, Vol. 53, p. 534, 1894. Aschkinass and Schaefer, Ann. der Physik, Vol. 5, p. 489, 1901.

[2] Blake and Fountain, Phys. Rev., Vol. 23, p. 257, 1906.

[3] W. B. Cartmel, Phys. Rev., Vol. 25, p. 64, 1907.

[4] W. R. Blair, Phys. Rev., Vol. 26, p. 76, 1908.

the intensity of the transmitted and reflected energy when electric waves are transmitted through thin plates. He considers the observed variations in intensity of both the transmitted and reflected energy in accord with the formulæ, but his study of the energy relations was very limited as he was more interested in the phase relations upon reflection.

About two years ago the writers began investigating the absorption of electric waves by several resonator screens in succession. Eleven screens were made by pasting tin-foil resonators on sheets of straw-board. These were placed one behind the other in a holder about half way between the vibrator and receiver. The screens were parallel to one another and inclined to the direction of propagation, making an incidence angle of 22.5°. The distance between them was approximately 5 cm. Measurements were taken of the energy transmitted by one screen, two screens, and so on, up to eleven. The energy fell off gradually for the first three screens but four screens were found to transmit about three per cent. more than three. For the next three screens, *i. e.*, the fifth, sixth and seventh, the energy again fell off gradually, but on placing the eighth screen in the holder no diminution in transmission was observed. On the contrary a slightly increased transmission was noted. When the distance between the screens was varied, these peculiar breaks in the orderly course of the absorption curve also varied, occurring for instance upon the addition of the fifth and tenth screens, or the third, sixth and ninth. Thus instead of building up a more or less homogeneous medium for electric waves, *i. e.*, one for which transmission varied regularly with the thickness, each screen of resonators acted like a new reflecting surface, and between each two screens "multiple reflections" took place, giving anomalous results which depended upon the combination of amplitudes and phase relations of the transmitted and reflected beams. The results were further complicated by the absorption of the medium. These phenomena, first observed in the attempt to solve a different problem, appeared sufficiently new and important to justify further study, so that the writers turned aside from their original purpose to seek a solution of these interesting interference phenomena complicated by resonance.

2. Apparatus and Methods of Measurement.

Vibrator.—The vibrator used was a special form of Righi vibrator,[1] chosen after several months' work with vibrators of the same general type, but of quite different construction. It was designed to meet the following needs: a vibrator which could be easily cleaned, easily and accurately reset at the focus of the collecting mirror after repolishing, and one which would have the smallest possible amount of metal and dielectric coming within the aperture of the

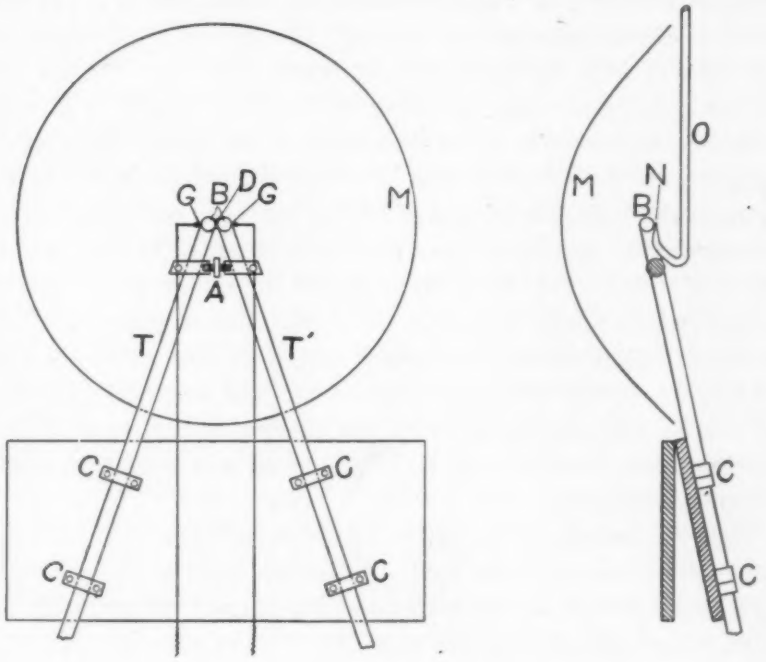


Fig. 1.

mirror. The essential details of construction are shown in Fig. 1. Two steel balls B, 0.95 cm. in diameter, rested in sockets accurately ground in the upper ends of two glass tubes TT', of one centimeter outside diameter. Below the aperture of the mirror M, these glass tubes were rigidly fastened to a plank by means of the hard rubber blocks C and about 10 cm. below the steel balls they were further stiffened by means of the hard rubber turn-buckle A. The screw

[1] Webb and Woodman, Phys. Rev., Vol. 29, p. 89, 1909.

of the turn-buckle was finely threaded and made a very satisfactory arrangement for adjusting the oil-gap. The turn-buckle also served as a convenient place for supporting the lead wires from the induction coil, which were bent at right angles just before coming to the air gaps *G* and terminated in small balls of copper made by fusing the ends of the wires in a Bunsen flame. The air gaps were about 3 mm. long and the spark took place along the axis of the vibrator. The source of energy was a 10-in. induction coil, with interrupter and condenser removed, connected to a 110-volt 60-cycle alternating current circuit. The current in the primary was usually kept at approximately seven amperes. The oil was fed through a glass tube *O*, ending in a capillary nozzle *N* directly in front of and slightly above the centers of the balls. By properly adjusting the flow of oil, a "film" *D* of any desired size would collect between the balls, the surplus of oil flowing down on the outside of the nozzle and dropping into a receptacle from the lowest point of the feed tube. The oil current served to dislodge any carbon bridges which might form due to the decomposing action of the spark as well as to keep a constant supply of fresh oil in the gap. As long as the film was kept constant and the length of gap small no trouble was experienced with the passing of the spark around the film instead of through it. Paraffin oil was used throughout these experiments.

The vibrator was set as nearly as possible at the focus of the parabolic mirror and then the final adjustments were made by setting up a bare receiver (one with no collecting mirror) 360 cm. from the vibrator and so adjusting the position of the vibrator that the distribution of energy in the reflected beam was symmetrical about the axis of the mirror in both the vertical and horizontal planes. This adjustment was tested from time to time, but it was not found necessary to reset the vibrator during a period of three months. In order to clean the vibrator it was only necessary to remove the balls, polish them and replace them in their sockets again, all of which could be done in less than five minutes without disturbing either the oil feed or the position of the vibrator with reference to the focus. This could be done at any time during a set of readings, as changing the vibrator usually made very little

difference in the ratios of the galvanometer throws, especially when working with tuned receivers. Such a vibrator as this gives a much more symmetrical beam than does a vibrator having an enclosed oil receptacle. This is probably due to the small amount of dielectric in contact with the balls, as well as the small amount of metal and dielectric in the aperture of the mirror. The symmetry of the beam from a vibrator of this type will be more fully discussed under the section dealing with the parabolic mirrors.

Receiver.—Both tuned and non-selective[1] receivers of the Klemencic type were used. In all receivers the wings were made of copper foil 0.15 mm. thick and 3 mm. wide and were fastened securely to a wooden base 12 mm. wide by means of wax, the space between them being 1 mm. The galvanometer lead wires were soldered to one side of the inner ends of the copper strips, while directly opposite to them on the upper side of these strips were soldered the fine wires of the thermo-junction. For the less sensitive receivers the thermo-couple was made of commercial iron and constantan wires 0.04 mm. and 0.025 mm. in diameter respectively, while for the more sensitive receivers platinum and constantan wires approximately 0.0045 mm. in diameter were used. The fine wires were soldered to the ends of the copper strips with a free end about a millimeter in length extending beyond the end of the strips. Under a binocular microscope these free ends were looped around each other with a spring contact and then either welded by means of an electric spark or soldered. The resistance of the thermo-junction was carefully determined before and after each set of observations and the sensitiveness was assumed to remain constant as long as there was no change in resistance.

In order to eliminate as far as possible the error due to the deterioration[2] of the vibrator and the consequent change in the energy emitted, a check receiver[3] was used, constructed as nearly like the main one as possible and the readings from both receivers were taken simultaneously. The ratio of the two galvanometer throws was always taken as proportional to the energy received by the main receiver.

[1] Webb and Woodman, *l. c.*

[2] Blake and Fountain, *l. c.*

[3] Klemencic and Czermak, Wied. Ann., Vol. 50, p. 175.

Several tuning curves were taken, some of which are plotted in Fig. 2. The total lengths of the main receivers are plotted as abscissae and the corresponding ratios of the galvanometer throws as ordinates. For convenience in plotting most of the ordinates have been reduced so as to make 100 the highest point on the curve. L_I shows a maximum at 23 mm., with an indication of a second maximum at 27 mm. In L_{II} the maximum occurs at 24 mm. while in L_{III} it comes at 25 mm. L_{IV} shows a decided maximum at 27 mm., L_V and L_{VI} have somewhat broader maxima, approximately at 24

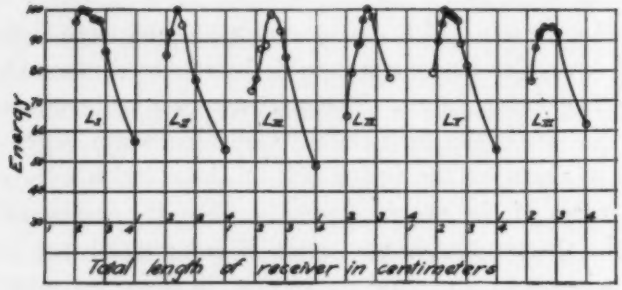


Fig. 2.

mm. in L_V and at 27 mm. in L_{VI}. From a study of ten or more such curves the conclusion is drawn that the vibrator emits a short spectrum, and that whether the maximum energy is obtained for a receiver length of 23 mm. or 27 mm. depends upon the characteristics of the receiver. This coincides well with the conclusion reached by Willard and Woodman,[1] that the form of the energy curves depends upon both receiver and vibrator. All of the receivers used were as nearly alike as possible, yet the distribution of the dielectric and the distance apart of the copper strips may have varied sufficiently to slightly change the resonance length of receivers of which the total length was only 25 mm. The fact that the vibrator cannot be considered a source of monochromatic radiation will be of considerable importance when we come to the discussion of the interference curves. Twenty-four mm. was chosen as the length of receiver to use in all work involving the use of a tuned receiver. The non-selective receivers were 310 mm. long, or approximately thirteen times the length of the tuned receiver.

[1] Willard and Woodman, PHYS. REV., Vol. 18, p. 3, 1904.

Parabolic Mirrors.[1]—The vibrator and receiver were each mounted at the focus of a copper parabolic mirror (paraboloid of revolution), silver-plated on the reflecting surface. Each mirror had an aperture of 56 cm., a depth along the axis of 15.2 cm. and a focal length of 12.2 cm. As most of the results in this paper are independent of any effect the mirrors may have had in determining the wave-length, no attempt was made to avoid "mirror action"[2] in the selection of these mirrors. Each mirror was mounted with its optic axis parallel to the floor and could be brought into proper alignment by rotation about either a vertical or a horizontal axis.

As the object in using these mirrors was to obtain a parallel beam of energy, it seemed worth while to test the beam for parallelism. This was attempted in three ways. The first consisted in comparing the actual distribution of energy in a plane perpendicular to the optic axis with that calculated on the assumption that the vibrator represented a point source placed at the focus of the mirror. The second consisted in comparing the distribution of energy perpendicular to the optic axis at a distance of 240 cm. from the mirror with the corresponding distribution at a distance of 360 cm. In a parallel beam the distribution at the two distances should be the same, provided the distances are sufficiently large to disregard the direct radiation. The third method consisted in taking readings with the main receiver placed at successive distances from the mirror along the optic axis. Unless the beam was diverging or the energy absorbed by the medium through which the waves passed, these readings ought to remain constant. In all three cases the distribution of energy was measured with a bare receiver (one with no collecting mirror), tuned to the incident energy and mounted in such a way that readings could be taken well outside the limits of the beam either horizontally or vertically, always starting from the center of the beam and taking the readings first on one side of the center and then on the other.

The theoretical distribution of energy from a point source placed at the focus of a paraboloid of revolution may be derived as follows:

[1] The mirrors were head-light mirrors, bought from the Adams and Westlake Company of Philadelphia.

[2] Webb and Woodman, *l. c.*

Let OP, Fig. 3, be a section of the paraboloid having its focus at F. About F as a center construct a sphere of any radius r.

Let E_s equal the energy per sq. cm. passing through the surface of the sphere. This will be a constant for a sphere of given radius.

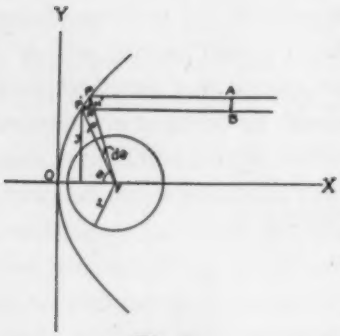


Fig. 3.

The total energy passing through the annulus on the sphere of width $d\theta$ and situated at an angle θ from the axis of the mirror is $2\pi r^2 \sin\theta d\theta E_s$. This same energy is contained in the hollow cylinder of thickness AB. The area of the cross-section of the hollow cylinder is $2\pi y dy = A_1$.

But $y = \rho \sin\theta$ where ρ is the radius vector of the paraboloid and $dy = AB = \rho d\theta$. Hence $A_1 = 2\pi y dy = 2\pi\rho^2 \sin\theta d\theta$.

Let E_p be the energy per sq. cm. passing through the plane AB. Then

$$E_p 2\pi\rho^2 \sin\theta d\theta = E_s 2\pi r^2 \sin\theta d\theta,$$

or

$$E_p = \frac{r^2}{\rho^2} E_s = \frac{r^2 E_s}{4p^2}(1+\cos\theta)^2 = K(1+\cos\theta)^2,$$

where p is the focal length of the parabola and K is a constant.

The theoretical curve can now be plotted by plotting as abscissæ the distances from the axis of the mirror perpendicular to the direction of propagation and for ordinates the intensities for the corresponding values of θ. Thus for abscissæ

$$\rho \sin\theta = \frac{2p\sin\theta}{1+\cos\theta} = K' \frac{\sin\theta}{1+\cos\theta}$$

and for ordinates

$$E_p = K(1+\cos\theta)^2.$$

For convenience in plotting E_p is taken as proportional to

$$\frac{(1+\cos\theta)^2}{4}.$$

The theoretical values are represented by the solid dots in Fig. 4, while the experimental values are represented by circles. Curves V_I, V_{II}, H_I and H_{II} represent the actual distribution of energy in the reflected beam. The abscissæ represent distances on either side of the center of the beam (the axis of the mirror), and the ordinates represent the energy expressed in per cent. of the energy at the center of the beam. It was found by trial that a metal screen having a circular opening 57 cm. in diameter placed 110 cm. from the vibrator cut off some of the direct radiation and hence gave a much more sharply defined *outside limit* to the beam. Most of the curves show that when the receiver had been moved 30 cm. from the center of the beam, which corresponds very closely with the half aperture of the mirror, the energy had decreased to approximately ten per cent. of its value at the center of the beam. For distances greater than 30 cm. the energy decreased very rapidly and at 40 cm. from the center of the beam barely five per cent. of the maximum energy could be detected.

V_I represents the vertical distribution at a distance of 360 cm. from the vibrator and H_I the horizontal distribution for the same distance. V_{II} and H_{II} are the corresponding distributions at a distance of 240 cm. from the vibrator. V_I and H_{II} correspond very closely with the theoretical distribution in a parallel beam. V_{II} and H_I do not agree so closely with the theoretical distribution, but even in these cases the agreement is remarkably close when one takes into consideration the fact that the vibrator is not a point source, as was assumed in working out the theoretical curve, and also the fact that the vibrator itself, together with its mountings, tends to disturb the symmetry of the beam. But perhaps the better test is to compare V_I with V_{II} and H_I with H_{II}. Such a comparison shows that the beam is slightly diverging, and also that the divergence is slightly greater in the horizontal plane than in the vertical. This difference in divergence in two planes at right angles to each other is probably due to the lack of symmetry of the vibrator, one distribution being measured along a line parallel to the direction of the electric force and the other along a line perpendicular to the electric force. The fact that the receiver is nearly ten times as long as it is wide would also tend to give different re-

sults as it is moved parallel to itself either horizontally or vertically across the beam.

Curve *A*, Fig. 4, shows the results obtained by moving the bare receiver along the axis of the mirror in the direction of propagation of the incident energy. Readings were taken at every ten centimeters between the two points used in determining curves V_{II}, V_{I}, etc., *i. e.*, between 240 cm. and 360 cm. from the vibrator. Distances from the vibrator are plotted as abscissæ and galvanometer

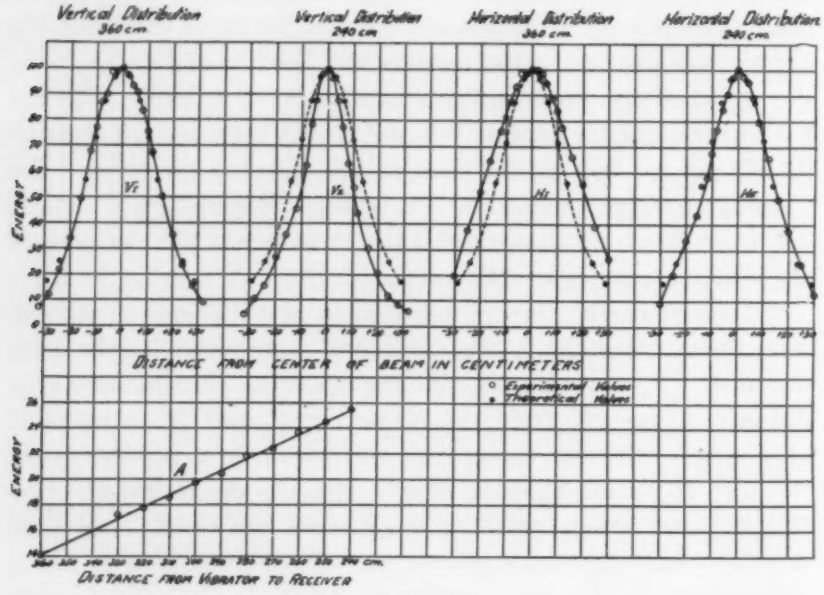


Fig. 4.

ratios, which are proportional to the energy, are plotted as ordinates. In moving the receiver a distance of 120 cm. towards the vibrator the energy nearly doubles, which would also indicate a diverging beam. The curve corresponds very closely to the distribution of energy in a beam diverging from a point source 110 cm. back of the vibrator, the intensity of the energy decreasing according to the inverse square law. If this inverse square relation were rigidly true, curve *A* could not be a straight line as plotted, but in any case such a limited portion of an inverse square curve taken at so great a distance from the origin would be very nearly a straight

line. The angle representing the total divergence of the beam is approximately ten degrees, in which case the error would be very slight in considering a portion of the wave surface 60 cm. long taken at a distance of 350 cm. from the source as a plane wave front. An attempt was made to vary the divergence of the beam by moving the vibrator on either side of the focus along the optic axis of the mirror, but practically no change was found in the distribution of energy unless the vibrator was moved a centimeter or more from the focus, and as such a change seemed too large for continued use the vibrator was finally set back at the geometrical focus and kept there for the remainder of the work.

The three tests for parallelism showed that the beam was slightly diverging and yet nearly enough parallel to introduce no serious error in the experimental results which follow.

Galvanometers.—The galvanometer used for measuring the energy received by the check receiver was of the du Bois-Rubens two-coil type, triply iron-clad. Previous investigators[1] had found it necessary to increase the shielding from magnetic disturbances by adding additional armor. This reduced the throw due to outside disturbances to so small a quantity that it was disregarded. The sensitiveness of this galvanometer was 4×10^{-9} volts. For measuring the energy of the main receiver was used a galvanometer of the type designed by Nichols and Williams,[2] with a sensitiveness of 2×10^{-9} volts. The half-period of each galvanometer was 2.8 seconds.

3. Interference Curves with Tuned Receivers.

Airy[3] has shown that the intensity of the light transmitted and reflected by thin plates or films may be computed from the following formulæ:

$$I_T = \frac{a^2(1-b^2)^2}{1-2b^2\cos\delta+b^4}$$

for the transmitted energy, and

$$I_R = \frac{4a^2b^2\sin^2\frac{1}{2}\delta}{1-2b^2\cos\delta+b^4}$$

[1] Blake and Fountain, *l. c.*

[2] Nichols and Williams, Phys. Rev., Vol. 27, p. 250, 1908.

[3] Sir G. Airy, Trans. Camb. Phil. Soc., Vol. 4, p. 419, 1830.

for the reflected energy, in which a is the amplitude of the incident ray, b the reflection coefficient when the reflection takes place in the rarer medium and δ is the phase retardation represented by the equation

$$\delta = \frac{2\pi}{\lambda} 2e \cos \omega,$$

λ being the wave-length in the material of the film, e the thickness of the film and ω the angle of refraction. These formulæ take no account of a possible phase change due to reflection or refraction and were therefore not directly applicable to the present problem. Professor R. C. Maclaurin assisted the writers in working out the following relations which more nearly express the conditions of the present investigation.

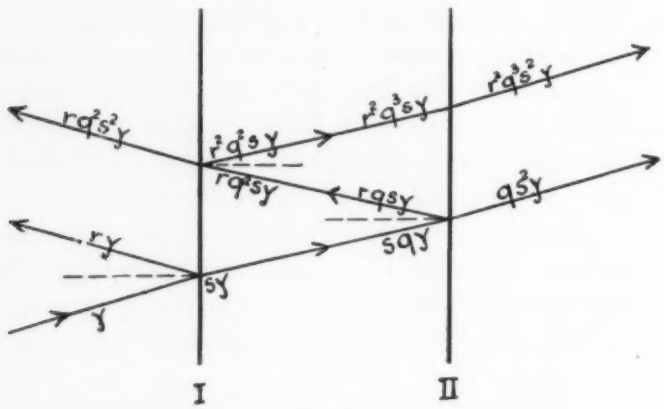


Fig. 5.

Let I and II, Fig. 5, be two screens of resonators placed parallel to each other a known distance apart. An incident ray y will give rise to multiple reflections between the screens and the first two transmitted and reflected rays will be as shown. The total energy in either the transmitted or the reflected beam will be equal to the sum of all these rays that emerge after one or more reflections or refractions, taking into account the amplitude and the phase of each ray at the time when the addition takes place. For the sake of showing the separate rays the figure has been drawn with the incident ray making a small angle of incidence, but in the deriva-

tion of the expressions for the intensities as well as in the experimental work the incident ray was normal to the reflecting surface.

We can represent the incident wave by the expression

$$y = A \cos 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} \right) = A \cos \theta,$$

equals the real part of $Ae^{i\theta}$; or, $y =$ the real part of $e^{i\theta}$, since we can assume the amplitude of the incident wave as unity.

Let $r = Be^{i\psi}$ be a complex number such that when it is multiplied by the expression for the incident wave it will give the amplitude and also the phase of the reflected wave. In case there is no absorption B is the amplitude of the first reflected wave.

Let $s = Ce^{i\phi}$ be a similar complex multiplier for the transmitted wave. In case there is no absorption in the medium and no loss of energy at the reflecting surface, $B^2 + C^2 = 1$.

Let $q = e^{-Kx} e^{i\alpha}$ be a complex quantity such that when multiplied by the expression for the transmitted beam at one surface, it gives the value of the transmitted beam just previous to the next reflection or refraction. K is the damping factor to take account of the loss of energy due to heating the resonators, scattering, etc., x is the distance between the screens, α is the phase change due to passing a distance x through the medium between the screens.

If y represents the incident wave,

ry will represent the first reflected wave,

sy will represent the first refracted wave at surface I,

qsy will represent the first refracted wave at surface II,

qs^2y will represent the first transmitted wave,

and so on.

The total transmitted wave will be represented by

$$y_t = qs^2y[1 + r^2q^2 + (r^2q^2)^2 + \cdots] = \frac{qs^2y}{1 - r^2q^2}.$$

Substituting the values given above for q, s, y and r, and writing $2\psi + 2\alpha = \delta$ we find

$$\frac{qs^2y}{1 - r^2q^2} = \frac{e^{-Kx} C^2 e^{i\alpha} e^{i(\theta + 2\phi)}}{1 - B^2 e^{-2Kx} e^{i\delta}} = Re^{i\beta},$$

where R is the amplitude of the transmitted wave and β is the phase angle.

The value of R is found to be

$$R = \frac{C^2 e^{-Kx}}{\sqrt{1 - 2B^2 e^{-2Kx} \cos \delta + B^4 e^{-4Kx}}}.$$

Therefore, since the intensity is proportional to the square of the amplitude,

$$I_t = K' R^2 = K' \frac{C^4 e^{-2Kx}}{1 - 2B^2 e^{-2Kx} \cos \delta + B^4 e^{-4Kx}}.$$

This reduces to the same form as Airy's formula if we put $K = 0$ and remember that $1 - B^2 = C^2$ on the assumption that the reflected energy plus the refracted energy is equal to the incident energy.

The expression for the transmitted energy will be nearly a maximum when

$$\delta = 2n\pi$$

and nearly a minimum when

$$\delta = (2n + 1)\pi,$$

In the expression

$$\delta = 2\psi + 2a,$$

2ψ represents the phase change due to reflection and would remain constant for any one wave-length and any given distribution of resonators on the resonator reflectors. a, on the other hand, is the change of phase due to the passage of the wave through the distance between the screens. If now the distance between the screens is varied continuously through several wave-lengths, a and hence δ will take all possible values between zero and $2n\pi$, and the energy transmitted by the two screens will increase and decrease through a regular succession of maxima and minima.

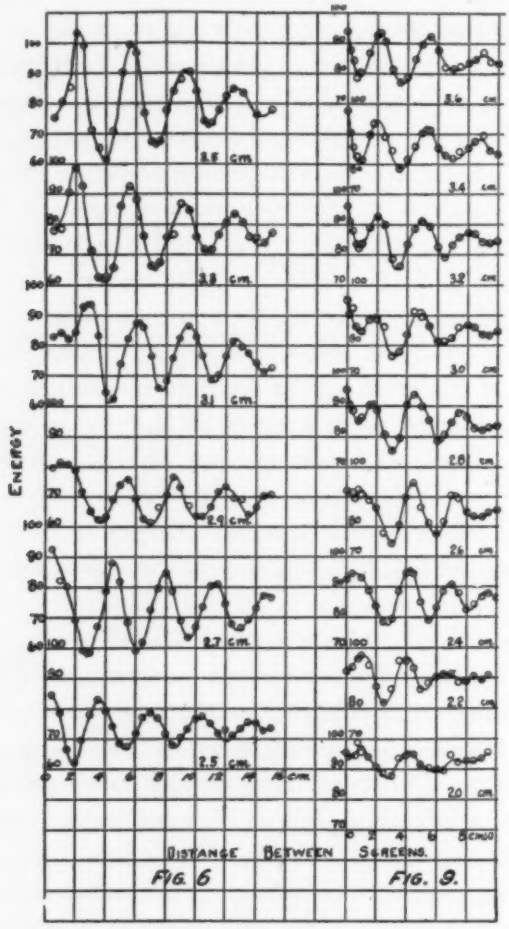
This was tested in the following way. Half way between the vibrator and the main receiver there was placed a frame for holding the screens of resonators perpendicular to the direction of propagation. The first screen was fixed 180 cm. from the vibrator, and the second was so mounted that its distance from the first could be varied through a range of 20 cm. or more. The screens of resona-

tors were made as nearly alike as possible. The resonators were pasted on straw-board nailed to wooden frames 86 cm. square, and consisted of strips of tin foil 2 mm. wide, and 3.5 cm. long, placed 1 cm. apart "end on" and 4 cm. apart "side on." On each screen were 18 columns and 21 rows.

The check receiver was 2.4 cm. long and was placed about 30 cm. directly in front of the vibrator. Owing to the lack of sensitiveness of the check and the lack of planeness of the reflecting screens, the effect of the reflected energy on the check was so small that it was disregarded. It was found necessary to change the position of the check in some later work with more sensitive receivers. The main receiver was also 2.4 cm. long and was mounted in the center of the beam 360 cm. from the vibrator. No collecting mirror was used. In order to further eliminate the errors due to change in ratios as the oil gap lengthened, alternate readings were taken on some standard condition and the condition under investigation. The average of the readings for any given distance apart of the screens was divided by the average of all the readings on the standard condition taken just before and just after the readings on the point in question. Inasmuch as the standard condition was a single screen, this quotient would represent the per cent. of the energy transmitted by the first screen alone that was transmitted by both screens.

The results with tuned receivers are plotted in Fig. 6. The distances between the screens are plotted as abscissæ. The ordinates marked 100 would mean that the same amount of energy was transmitted by two screens as by one, whereas the ordinate marked 60 would mean that two screens transmitted sixty per cent. of the energy transmitted by one alone. Six curves were taken, showing the results obtained with resonators of lengths 3.5 cm., 3.3 cm., etc., to 2.5 cm., the length for resonance being 2.9 cm. The curves all show very decided maxima and minima, which for the most part fall at regular intervals, the average distance between successive maxima or minima being 3.5 cm. This agrees fairly closely with the half wave-length of the incident energy, which is probably either 3.1 cm. or 3.2 cm. If the vibrator was emitting anything like monochromatic radiation we would expect the distance between

the successive maxima or minima to equal the half wave-length in air, but when we remember that the wave-length determination is at best only an average of a very complex emission, and that the interference curves are taken under conditions where we get interference between certain groups of wave-lengths at a time depend-



Figs. 6 and 9.

ing on the distance between the reflecting surfaces, similar to the interference of white light when falling on a film of oil of varying thickness, it does not seem so remarkable that the half wave-length does not check more closely than ten per cent.

It is interesting to note in Fig. 6 the change of curvature within the first two centimeters as the length of the resonators is changed. The first two curves start out with a very decided upward slope, while the last two have a downward slope in the same region, the change apparently occurring at or near the resonance length of the resonators. We may regard this merely as a shift of the maxima and minima to the right or the left. The position of the first maximum is undoubtedly determined in part by the phase change due to reflection. Although the curves do not admit of accurate numerical calculations of this phase change, it seems justifiable to conclude that the change in position of the first maximum indicates that the phase change due to reflection is not a constant for all lengths of resonators. On the contrary, it takes different values as the length of the resonators is varied, at least when the lengths are taken near the resonating length.

The change in the per cent. of energy transmitted by two screens becomes less and less as the distance between the reflecting surfaces increases. By the time the distance between the screens has become equal to two wave-lengths, the per cent. of energy transmitted by two screens has become nearly constant, being about seventy or eighty per cent. of the energy transmitted by the single screen. This apparent decrease in the energy is probably due partly to the complexity of the radiation and partly to the loss of energy by absorption in the resonators themselves as well as by scattering.

4. Interference Curves with Non-Selective Receivers.

(*a*) *By transmission.*—The arrangement of the apparatus for the interference curves obtained by transmission through two screens of resonators with non-selective receivers is given in Fig. 7. M_1 and M_2 are the parabolic mirrors, the vibrator, V, being at the focus of M_1 and the main receiver, MR, at the focus of M_2. The resonator screens were placed at S, halfway between the vibrator and the Boltzmann mirrors, BM. The energy transmitted by the resonator screens was reflected from the Boltzmann mirrors in the direction of M_2, the angle between the incident and reflected rays being 20°. S_2S_2 show the position of the screen used to cut off part of the direct radiation and so make the distribution of energy in the

reflected beam correspond more closely to that in a parallel beam. At S_1 was placed a small zinc screen covering up the lower portion of the aperture of M_1. It was found that with sensitive receivers the energy of the check would increase nearly twenty per cent. due to the energy reflected back from the resonator screens. As the focus of the parabolic mirror was inside the plane of the aperture no position for the check could be found where it would not receive this reflected energy and still receive direct radiation from the vibrator. By placing the screen at S_1 a position could be found for

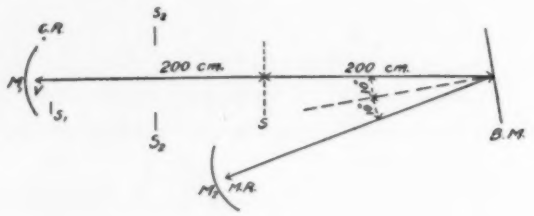


Fig. 7.

the check receiver, CR, where the readings of the check did not vary when readings were taken with and without the screen S in position.

The Boltzmann mirrors were placed in the position indicated, in order that wave-length determinations could be made using the energy which had been transmitted by the screens. For the interference curves the two mirrors were set in the same plane and used simply as a metal reflector. Each of the mirrors was made of plate glass, 76 cm. wide and 38 cm. high, covered with tin foil. They were mounted on vertical wooden frames with horizontal arms which in turn were made to slide on cross pieces on a rigid wooden frame. A more detailed description of the Boltzmann mirrors may be found in an earlier paper by the writers.[1]

The resonators were pasted on tracing cloth which had been moistened and stretched as tightly as possible over an artist's frame for stretching canvas. Two screens of resonators were pasted, each containing eighteen columns and twenty-nine rows. The resonators were 3.6 cm. long at the start, 3.0 cm. apart "side on" and 1.0

[1] Webb and Woodman, *l. c.*

cm. apart "end on." The resonators were shortened 0.2 cm. each time until the final length was 2.0 cm.

The non-selective receiver was chosen for this set of interference curves on account of the complex character of the energy given out by the vibrator. In fact the expression "tuned receiver" has very little meaning when used in connection with energy that has been transmitted through one or more screens of resonators. A receiver that is in tune with the incident energy without the screen of resonators is no longer in tune with the energy that has come through the screen. And in work of this kind where several different lengths of resonators are used a receiver of fixed length cannot possibly be in tune with the energy transmitted through the various lengths of resonators. This conclusion is based partly on a study of tuning curves taken with resonator screens placed in the path of the beam, but largely upon a study of the wave-length curves to be given later.

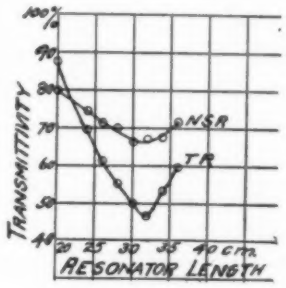


Fig. 8.

A comparison of the results obtained with the two types of receivers is given by the curves in Fig. 8. The abscissæ represent the lengths of resonators in centimeters and the ordinates represent the per cent. of the incident energy which is transmitted by the screens on which were various lengths of resonators. The curve marked *TR* gives the results obtained with a tuned receiver, while the curve marked *NSR* gives the results with the non-selective receiver. The most striking difference is the flatness of the curve taken with the non-selective receiver in comparison with the sharp minimum of the curve taken with the selective receiver. The latter very closely resembles the curves obtained by Blake and Fountain,[1] and would give 3.2 cm. as the length of resonator for a minimum of transmission and a maximum of reflection. While the curve taken with the non-selective receiver is too flat to admit of very great accuracy in locating the minimum of transmission, yet it seems to agree very closely with that obtained with the tuned receiver.

[1] Blake and Fountain, *l. c.*

Either receiver would measure the same per cent. of the incident energy when this energy has been transmitted through resonators 2.2 cm. long. Undoubtedly the curves would also have crossed for some length of resonator longer than the resonator length, but no attempt was made to find this length. From a study of such curves as these it seems to the writers that for work of this kind the non-selective receiver has very marked advantages over the tuned receiver, expecially when one is dealing with energy as complicated as that emitted by a Righi vibrator. The chief objection to the use of the non-selective receiver lies in the fact that the ratios of the energies measured by the two receivers decrease very rapidly as the oil gap becomes foul. This difficulty was overcome in a very satisfactory manner by adopting some standard condition with which readings could be taken as often as desired. For most of the work five readings were taken on the standard condition and four on the condition under investigation, always beginning and ending with the standard condition. The five readings on the standard condition were then averaged and also the four on the condition under investigation. Finally the ratio of the two averages was taken for the basis of comparison as well as for use in plotting the curves. These ratios usually agreed within two per cent., in which case only two sets of observations were taken for any given set of conditions, but in case the disagreement was more than two per cent. three and sometimes four sets of observations were taken.

The transmission interference curves taken with the non-selective receivers are given in Fig. 9. For the purpose of comparison they are plotted to the same scale as those obtained with the tuned receivers, the numbers just below each curve being the resonator length in each case. Most of the curves show well-defined maxima and minima, the average distance between two successive maxima or minima being 3.2 cm., which is a very close agreement with the half wave-length as determined with non-selective receivers. The total change in energy between the highest maximum and the lowest minimum is usually less than twenty per cent. of the energy transmitted through one screen, whereas with the tuned receiver it sometimes amounted to as much as forty per cent. The energy approaches a constant value much more rapidly than with the tuned

receivers, so that it was useless to increase the distance between the screens beyond 10 cm. The last curve taken with a resonator length of 2.0 cm. is very flat and the maxima and minima disappear when the distance between the screens is only six centimeters. For the first seven curves the constant value towards which the energy approaches was approximately 85 per cent. of the energy transmitted by the single screen. However, when the resonators are 2.2 cm. long, this constant value is 90 per cent., and for resonators 2.0 cm. long it is approximately 92 per cent. This means in the last curve that the two screens placed six centimeters or more apart will transmit 92 per cent. of the energy that would be transmitted by a single screen, whereas, if the resonators were 2.6 cm. long, the same two screens placed 10 cm. or more apart would transmit 85 per cent. of this energy. A comparison of the positions of any particular maximum or minimum, like the second or third maximum for instance, in the curves taken in order down the page shows a gradual shifting of the position of the maximum towards the left. This shift is probably due to the fact that the change of phase due to the reflection gradually changes as the resonators are shortened. In some of the curves certain irregularities occur within the first centimeter or two. These are probably due partly to experimental errors and partly to other causes for which no adequate explanation can be given.

One set of readings was taken using three screens of resonators. The two screens nearest the vibrator were kept at the constant distance apart of 4.5 cm. This distance was chosen because with it the energy transmitted by the two screens was neither a maximum nor a minimum, in fact it corresponded to a point on the axis of the interference curve. The resonators on each of the screens were 3.6 cm. long and were in all respects like the screens already described for the work with the non-selective receivers. The distance between the second and third screens was increased from zero to six centimeters, usually adding 0.5 cm. each time, and the energy transmitted by the three screens determined. The energy passed through a series of maxima and minima as for the two screens. Two successive maxima or minima were found to be 3.2 cm. apart, and the total change in energy amounted to about fifteen per cent. of

the energy transmitted through the two screens, which was taken as the standard for comparison.

(*b*) *By Reflection.*—The arrangement of the apparatus for the work by reflection was very similar to that ordinarily used for wave-length determinations with the interferometer.[1] The separating surface[2] consisted of a wooden frame 101 cm. by 96.5 cm., on which were stretched strands of no. 34 copper wire spaced one centimeter apart. The frame was mounted with the wires horizontal, or parallel to the electric force. The energy reflected from this wire grid agreed with the transmitted energy within less than two per cent. The movable mirror was a piece of plate glass 76 cm. by 77 cm. covered with tin foil. It was mounted in the same position as that occupied by the resonator screen in the work on transmission, and could be replaced at any time by the resonator screen. The distance between the vibrator and the movable mirror was 200 cm. while the distance between the vibrator and the center of the wire grid was 95 cm. The fixed mirror was also of plate glass covered with tin foil, 76 cm. square, and was placed 105 cm. from the center of the wire grid. The distance between the main receiver and the center of the wire grid was 150 cm.

For determining the interference curves the fixed mirror was replaced by a large metal screen supported in such a way that the energy falling upon it was deflected to one side and produced no effect upon the receiver. The movable mirror was removed and the two tracing cloth screens already described were put in its place. The position of the check was not changed from that shown in Fig. 7. The curves marked *R* in Fig. 10 show the results obtained by reflection. The screen nearest the vibrator was kept in the same position throughout the observations and the energy reflected from it was used as a standard condition. Four readings were taken with two screens a known distance apart alternating with five readings on the standard cond tion, always beginning and ending with the standard condition. Then the readings of the energy reflected from one screen and those of the energy reflected from two screens were averaged separately and the latter average divided by the

[1] Willard and Woodman, *l. c.* Blake and Fountain, *l. c.*

[2] G. F. Hull, PHYS. REV., Vol. 5, p. 231, 1897.

former. These ratios are plotted as the ordinates in Fig. 10, and the distances between the screens are plotted as abscissæ. For the purpose of comparison, the corresponding curves by transmission are plotted just below the curves by reflection. The two curves, *i. e.*, either reflection curve with its corresponding transmission

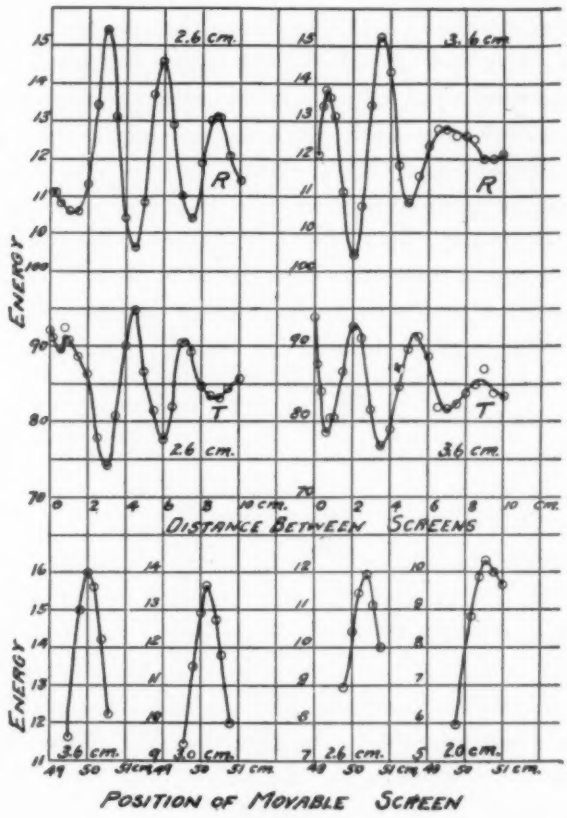


Fig. 10.

curve, are not plotted to the same scale on the Y-axis and no attempt was made to compare the absolute energy in the two beams. The same scale is used throughout for the abscissæ, so that the curves can be compared with reference to the position of the maxima and minima. Such a comparison shows that aside from a few irregularities within the first two centimeters, the corresponding curves agree very closely with each other, the maxima on the reflection curve

falling at the same points as the minima on the transmission curve and vice versa. This also agrees well with the deductions which can be drawn from Airy's formulæ for the intensity of the reflected and transmitted energy. The resonators used in determining the two curves plotted were 2.6 cm. and 3.6 cm. long respectively.

The phase change due to reflection was studied as follows: The fixed mirror was replaced in its proper position and one of the screens of resonators was used in place of the movable mirror. Readings were taken around what would correspond to the center maximum on the ordinary wave-length curve as determined with the interferometer. If there is no phase change due to reflection the position of the center maximum should not change whether the readings are taken with the movable mirror in that arm of the interferometer, or with the screen of resonators substituted in place of it. Furthermore the position of the center maximum should not change for different lengths of resonators on the screen. The results of these tests are given in the four curves at the bottom of Fig. 10, from which it is seen that there is a gradual shifting of the center maximum as the resonators are shortened. By shortening the resonators from 3.6 cm. to 2.0 cm. the position of the center maximum has been shifted 6 mm., which would correspond to the phase change produced in the wave while going a distance of 1.2 cm. This corresponds very well with the variation of the phase change due to reflection as shown in the interference curves obtained by transmission. See Fig. 9. The position of the center maximum for resonators 2.0 cm. long was just the same as for the movable mirror, which would seem to indicate that the phase change due to reflection for resonators of this length was zero. The whole question of phase change upon reflection should be more carefully investigated than the writers have attempted to do in this work.

5. WAVE-LENGTH DETERMINATIONS.

Another factor that must be taken into consideration when sending the complex radiation from a Righi vibrator through the screens of resonators is the *selective* reflection which takes place, especially if the resonators are near the resonating length. The phenomena which take place as the length of the resonators is gradually short-

ened through the resonating length may be easily understood, at least qualitatively, from a consideration of Fig. 11. The continuous curve represents an assumed distribution of energy in the short spectrum of a Righi vibrator, the wave-lengths being plotted as abscissæ and the corresponding intensities as ordinates. The dotted curves represent the assumed transmission curves of a screen of resonators for five different lengths of resonators, the curve *C* corresponding to the resonating length. The same coordinates are used for these curves as for the continuous curve. When the resonators are of the length corresponding to curve *A* they will transmit the energy from the vibrator with practically no change of wave-length, by which is meant the wave-length as determined with the Boltzmann mirrors or the interferometer. But when the resonators are of the length corresponding to *B* they will transmit much more of the shorter wave-lengths than of the longer and the resultant wave-length as measured by the Boltzmann mirrors and a nonselective receiver will be shorter than that measured without the screen. When the resonators correspond to *C* they will transmit the shorter and the longer wave-lengths of the spectrum nearly equally but the intermediate wave-lengths with much less intensity. Now the wave-length determination will no longer give a smooth curve, especially around the second maxima. We would expect that such a combination of wave-lengths as this would give a very broad second maximum on either side of the center maximum. But when the resonators correspond to *D* they will transmit more of the longer wave-lengths than of the shorter and the resultant wave-length will accordingly be longer than that obtained without the screen. Resonators of the length corresponding to *E* transmit all of the wave-lengths represented in the characteristic curve of the vibrator with practically no change in relative intensity.

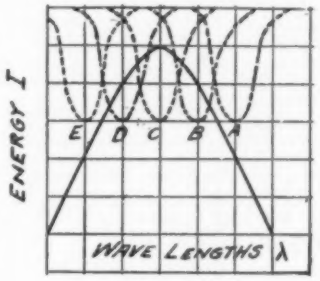


Fig. 11.

That this reasoning is correct, at least qualitatively, is clearly shown by the results plotted in Fig. 12. The arrangement of the

apparatus was the same as that shown in Fig. 7, and used for the interference curves by transmission with non-selective receivers. For each length of resonators used in obtaining the interference curves a wave-length curve was taken by means of the Boltzmann mirrors. The non-selective receivers were also used for the wave-length determinations, thus doing away with the many errors[1] in-

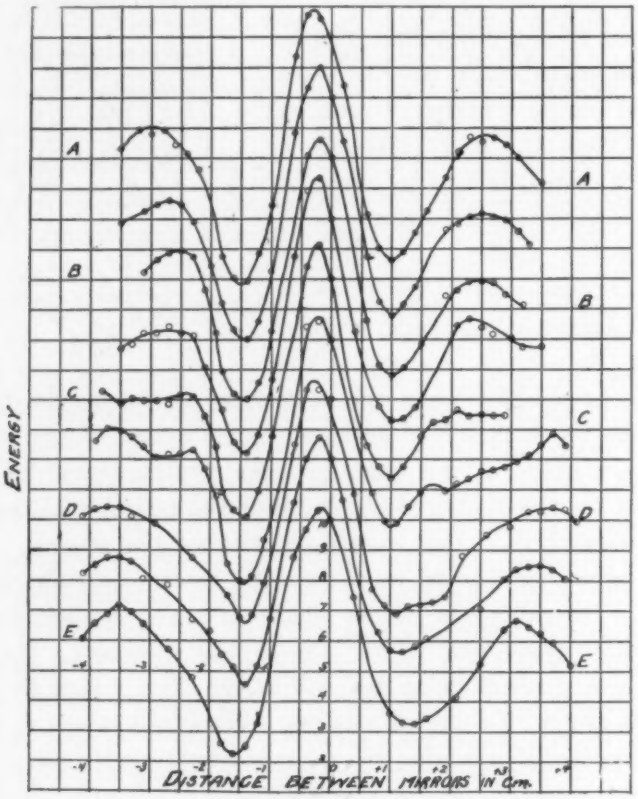


Fig. 12.

troduced by the use of selective receivers in wave-length work. For the curves given in Fig. 12 the energy was transmitted through one screen of resonators. The distances between the two Boltzmann mirrors are plotted as abscissæ and the corresponding intensities as ordinates. The readings were taken with reference to some position of the mirrors taken as a standard, a reading being taken for the standard position, then on the point in question, and finally

[1] Webb and Woodman, *l. c.*

on the standard again. The standard position was usually taken as near the center of a maximum or a minimum as possible, and then the ratio of the readings on the point in question to the average of the readings on the standard taken just before and after the point in question was computed. Then these ratios were multiplied by the proper factor to bring them into the proper relation to the center maximum, so that, as plotted, any ordinate represents the energy for the corresponding abscissa with reference to the energy received by the receivers for zero difference in path.

One great error in work of this kind arises from the fact that the characteristic curve of the emission of the vibrator changes as the spark gap deteriorates. This changes the selective action of resonators of a given length during a long set of observations and tends to make the wave-length curves irregular. For this reason the writers have made no attempt to use the curves in Fig. 12 for other than qualitative comparisons. The action of the receiver mirror as well as the comparatively large angle of incidence at the Boltzmann mirrors also tend to make the results unreliable for quantitative measurements, but do not make them any the less valid for the qualitative comparison intended by the writers. The curves in Fig. 12 are lettered to correspond at least roughly to the dotted curves in Fig. 11. The shortening of the wave-length as the resonators are shortened is very clearly marked in the first three curves. The next two show a flattening of the second maxima with the maximum energy on the side of the shorter wave-lengths, while the next two show the same flattening of the second maxima with the maximum energy on the side of the longer wave-lengths. In the last two the second maxima have become much sharper and indicate a wave-length longer than the true wave-length of the vibrator and gradually decreasing to the true wave-length.

Applying the same reasoning to the reflected energy one would expect that the wave-length of the reflected energy would be shorter than the true wave-length of the vibrator for the same length of resonators that gave a lengthening of the transmitted wave-length. This was tested by means of the interferometer arrangement already described in connection with the interference curves by reflection and found to be the case. The screen of resonators was used

as the movable mirror in the interferometer and the wave-length curves taken in the usual way. For resonators 3.6 cm. long the wave-length by reflection was lengthened about ten per cent. of the true wave-length, while for resonators 2.0 cm. long it was shortened about the same amount. The curves are not given as they are very similar to those of Fig. 12.

6. RESONATORS ON GLASS.

Further data bearing upon multiple reflections was obtained by substituting a large piece of plate glass for the tracing cloth screens. The two reflecting surfaces in this case were the two surfaces of the glass plate. The glass surfaces themselves reflected a large per cent. of the incident energy, but the reflection coefficients of one or both surfaces could be varied by the addition of resonators of different lengths. The plate was 96 cm. by 122 cm. and was placed in the beam perpendicular to the direction of propagation. The vibrator and receiver were placed 360 cm. apart and the plate was arranged on a carriage so that readings could be taken with the plate in and out of the beam alternately. The readings without the plate in the beam were used as a standard, so that the ratios of the readings "in" to the readings "out" give us the transmittivity of the plate under the given condition. The readings were taken with both tuned and non-selective receivers.

The results are plotted in Fig. 13. Resonator lengths are plotted as abscissæ and the transmittivity as ordinates. Curves *I* and *II* were taken with a glass plate 0.8 cm. thick, curve *I* being taken with resonators pasted on only the surface of the glass towards the vibrator, and curve *II* with the resonators on both surfaces, those on one surface being directly opposite those on the other. In curve *I* the reflection coefficient for one surface of the glass was constant, while that for the other surface varied as the resonators were shortened. In curve *II* the reflection coefficients for both surfaces were changed whenever the resonators were changed. The resonators were 3.0 cm. long at the start, 0.5 cm. apart end on, and 3.0 cm. apart side on, and there were thirty rows and twenty-three columns. The curves marked *NSR* were taken with the non-selective receiver, those marked *TR* with the tuned receiver. The curves with the non-selective receiver tell us very little about the length

of resonators for maximum resonance. Apparently the minima fall at the same places as those for the tuned receiver. With the tuned receivers the minima of transmission at the resonance length fall at 1.5 cm. for both curves *I* and *II*, showing that the addition of the resonators to the other surface of the glass did not change the length for resonance. It did slightly change the form of the curve, but more particularly the per cent. of the original energy that was transmitted. The transmittivity is in general less for the plate having the resonators on both sides. This may be due in part to

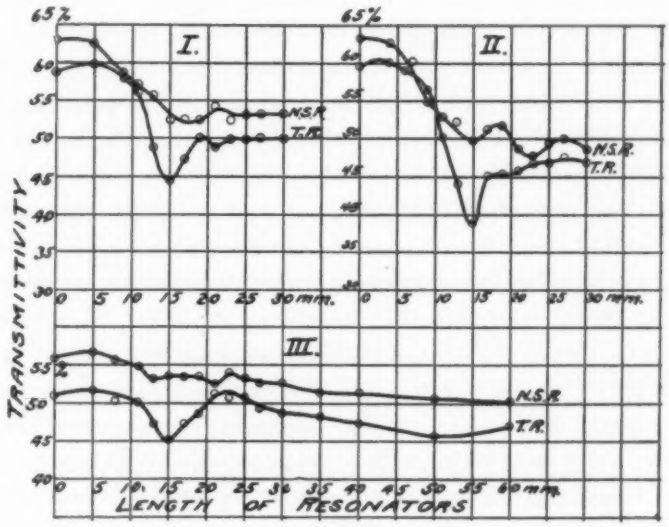


Fig. 13.

increased scattering, but is more probably due to the different combinations of reflection coefficients and phase changes due to reflection in the two cases. It is interesting to note that for resonators approximately 1.0 cm. long the transmittivity is the same for either type of receiver.

Curve *III* was taken with a plate of glass 0.6 cm. thick. The resonators were 6.0 cm. long at the start, 3.0 cm. apart side on and 1.0 cm. end on, and there were thirty rows and twelve columns. Very little need be said about the curve taken with the non-selective receiver, as the total change in the transmittivity was so slight that it made accurate work with such a receiver very difficult. The curve is published merely to show that the change in the total

energy is very much less marked than the change for a particular group of wave-lengths in resonance with a tuned receiver.

If we compare curve *I* with curve *III*, we see that the transmittivity for bare glass depends upon the thickness of the glass. For the plate 0.6 cm. thick the transmittivity was 51 per cent. of the incident energy, while for the plate 0.8 cm. thick the transmittivity was 59 per cent., using the tuned receiver curves for the basis of comparison. This agrees well with what we might expect from the results of the work on the transmission of two screens of resonators on tracing cloth. If we should measure the transmittivity of a large number of glass plates of different thicknesses, we would undoubtedly find that the transmittivity would pass through a succession of maxima and minima, only of course in this case the results might be further complicated by the possible absorption of the glass. In all probability the thicker of the two plates used absorbed more than the thinner plate and yet it transmitted 8 per cent. more of the incident energy than the thinner plate.

There is no evidence of *extra-transmission* in either curves *I* or *II*, unless it be for resonators less than 1.0 cm. long. For resonators 0.5 cm. in length the transmittivity is apparently one or two per cent. higher than for bare glass. Of course it is difficult to measure to this degree of accuracy and the apparent increase in transmittivity may be due to experimental error, but the readings were repeated several times and the higher transmittivity always came when the resonators were on the glass. The fact that it occurs in both curves *I* and *II* makes it seem more probable that it is a case of *extra-transmission*. In curve *III* the transmittivity for the plate when the resonators were 2.3 cm. long was equal to that for the bare glass, which is only a special case of *extra-transmission*. As this curve was taken with the same plate and with the same distribution of resonators as that used by Blake and Fountain[1] it might be interesting to compare the curve with the tuned receiver with the corresponding curve (curve *T*, Fig. 9) in their paper. It must be borne in mind that their results were obtained with 10.0 cm. waves, while the wave-length used by the writers was only 6.0 cm. approximately. The length for resonance was 2.55 cm. for the 10 cm. waves and 1.5 for the 6.0 cm. waves. They found the greatest extra-

[1] Blake and Fountain, *l. c.*

transmission when the resonators were twice the resonating length, while the length for *equal transmission* in curve *III* is approximately 1.5 times the resonating length. This seems to strengthen the idea of Blake and Fountain that the *extra-transmission* does not always come for lengths twice the resonating length but may come for "any length greater than the fundamental length." The writers are of the opinion that it may also occur for lengths shorter than the resonating length. In fact in several of the curves published by Blake and Fountain the transmittivity is the same as that for bare glass when the resonators were still a centimeter or more in length, and it is an interesting conjecture as to what form the curves would have taken if the readings had been taken for lengths shorter than one centimeter. Comparing the two curves further we notice that the change in transmittivity is very much less for the 6.0 cm. waves than for the 10.0 cm. waves, varying from 45 per cent. at the minimum to 51 per cent. at the highest point for the 6.0 cm. waves and from 38 per cent. to 76 per cent. for the 10.0 cm. waves. Also the transmittivity of the bare glass was different for the different wave-lengths, being 51 per cent. for the 6.0 cm. waves and 60 per cent. for the 10.0 cm. waves. That this was not due entirely to a greater absorption of the shorter wave-lengths is well brought out in the following paragraph.

Using the adjustable system of rod vibrators described in another paper by the writers[1] and for which the wave-lengths were fairly accurately determined, the transmittivity of a plate of glass was measured for six different wave-lengths. For this purpose the plate of glass 0.8 cm. thick was used with no resonators on either side. It was mounted perpendicular to the direction of propagation of the incident energy and could be easily moved in or out of the beam. The energy without the plate in the beam was again used as a standard, the readings being taken alternately "out" and "in," always beginning and ending a set of readings with the standard condition. Readings were taken with both the tuned and non-selective receivers, the length of the check receiver as well as that of the main being changed each time to correspond to the wave-length used. The length of the non-selective receiver, was kept constant at 60.0 cm. The results are given in the accompanying table.

[1] Webb and Woodman, *l. c.*

Total Length of Vibrator.	Receiver Length (Tuned Receiver).	Approximate Wave-Length.	Per Cent. Transmitted.	
			Tuned Receiver.	Non-selective Receiver.
6.55 cm.	6.60 cm.	15.7 cm.	56.8 per cent.	68.2 per cent.
5.05	5.05	12.1	64.5	64.0
3.78	3.80	9.1	53.7	55.0
3.21	3.20	7.7	41.0	53.4
2.50	2.50	6.0	50.5	61.7
2.00	2.00	4.8	69.0	—

It will be seen that for the tuned receiver the transmittivity varies from 41 per cent. to 69 per cent. for different wave-lengths. If the change had always been either an increase or a decrease, it might have been ascribed to the varying absorption by the glass of different wave-lengths. But if it had been due to absorption we would expect to get the greater absorption and hence the smaller transmittivity for the shorter wave-lengths, but this is clearly not the case. The fact, too, that the thickness of the plate is, even for the shortest wave-length, only one sixth of a wave-length makes the explanation based upon absorption seem very improbable. As one would expect, the variation in the transmittivity for the non-selective receiver is much less than for the tuned receiver. Here, too, the transmittivity decreases for the first four wave-lengths tried and then increases again as the wave-lengths are made still shorter. These results all show that the transmittivity of a plate of glass of given thickness and dielectric constant passes through a series of maxima and minima for varying wave-lengths, so that in specifying the transmittivity of a given plate of glass it is always necessary to specify for what wave-length the transmittivity is found.

7. CONCLUSIONS.

As a result of the present study it appears:

1. That a Righi vibrator of small dimensions, mounted with no oil holder surrounding the balls and with as little dielectric as posposible in front of the aperture of the mirror and placed at the focus of a paraboloid of revolution acts very nearly like a point source and gives very nearly a parallel beam of energy.

2. That the tuning curves as well as the wave-length curves show that the radiation from a Righi vibrator consists of a short spectrum

of wave-lengths and that the maximum on the tuning curves is governed within the limits of this spectrum by the peculiarities of the receiver.

3. That the non-selective receiver is far superior for all work involving the determination of wave-lengths or the tuning of resonators pasted on screens and placed in the path of the incident energy.

4. That the intensity of the transmitted and reflected energy passes through a series of maxima and mimina as the distance between the reflecting surfaces is increased through a small number of wave-lengths. This agrees well with the theory as worked out for optics.

5. That a screen of resonators acts like a selectively reflecting surface for waves very nearly in resonance with the resonators. This was tested by determining the wave-length of the transmitted beam as well as of the reflected beam. These were found to check at least qualitatively with what one would expect from a consideration of an assumed emission curve for the vibrator combined with an assumed transmission curve for the screen of resonators.

6. That with a tuned receiver *extra-transmission* may be expected for plates of glass of the proper thickness and of the proper dielectric constant and for wave-lengths within proper limits.

7. That the transmittivity of a glass plate of a given thickness and given dielectric constant depends upon the wave-length of the incident energy.

8. That the change of phase due to reflection is not a constant but varies with the length of the resonators pasted on the reflecting surface.

In conclusion the writers wish to thank Mr. H. W. Farwell and Dr. L. B. Morse for kindly assistance in taking some of the observations. They also wish to acknowledge their indebtedness to Professor M. I. Pupin who gave the services of his assistant, Mr. W. E. Cushman, during an illness of one of the writers. They are indebted further to Professor E. F. Nichols who suggested the problem and many details of the method for solving it.

PHŒNIX PHYSICAL LABORATORIES,
COLUMBIA UNIVERSITY, February, 1909.

THE ABSORPTION OF SODIUM VAPOR.

BY GEO. B. CLINKSCALES.

HISTORICAL.

THE fluted absorption spectrum of sodium vapor was first observed by Roscoe and Schuster in 1874. Subsequent investigations were made by Liveing and Dewar in connection with their work on the reversal of lines of metallic vapors, and also by Wood. Nine bands were photographed in the green and some others were observed visually in the red region. These bands they found consisted of a great number of fine lines, more or less regularly spaced along the region of absorption.

The fluorescence of the same vapor, excited by white light, was studied by Wiedemann and Schmidt[1] in 1896.

In 1904 Wood and Moore[2] made an investigation of both the absorption and fluorescence, endeavoring to find how the two were related, also to determine the effect of changing the wave-length of the exciting light. They proved conclusively that the two are in every way complementary, *i. e.*, an absorption band in one corresponds in position to a fluorescent band in the other. The presence of hydrogen was found to magnify greatly the appearance of the fluorescence, and to cause its disappearance when the pressure was above a few centimeters. Since these phenomena are so closely related it is perfectly natural to suppose that a study of the effects of pressure upon absorption will throw some light upon the mechanism of fluorescence.

Quite recently A. Dufour[3] has found that the absorption of bromine vapor and some other gases is greatly modified by the presence of some chemically inert gas above atmospheric pressure, and although fifty or more lines were found to shift, others were not.

[1] Wied. Ann., VII., 447, 1896.
[2] Phil. Mag. (6), 1904.
[3] Compt. Rendus, 145, 757–758, 1907.

Dufour investigated separately, and under the same conditions the effect of hydrogen, CO and air, all producing the same effect. This points to a purely mechanical effect as distinguished from a chemical one.

Wood in 1906[1] found that the absorption spectrum of sodium vapor is fluted in the presence of hydrogen at atmospheric pressure and above, but that at very low pressures the fluted form entirely disappears. These observations were made under comparatively low dispersion; and at the suggestion of Professor Wood the investigation to be described was begun in order to study the same phenomena with apparatus having higher dispersion.

The effects which will be considered in this paper should not be confused with the usual "pressure shifts" studied by Humphreys. It is probable that they are more nearly analogous to the phenomena described by Wood in the case of mixtures of mercury vapor with hydrogen, nitrogen, helium and other chemically inert gases in varying proportions.[2]

In this paper it was shown that the shape and even the position of an absorption band is greatly changed by the presence of a foreign gas at low pressures. It was first noticed when the vapor was in the presence of hydrogen, and thinking that it was due to some sort of chemical combination, he repeated the same experiment with nitrogen and helium. The effect was the same. There is much evidence brought out in this investigation that certain of the lines of sodium behave in a similar manner. The pressures are in most cases between 1 mm. and 760 mm., far below the pressure at which any true "pressure-shift" can be observed.

Object of the Research.

1. To observe and record the fine structure of the absorption spectrum of sodium vapor.

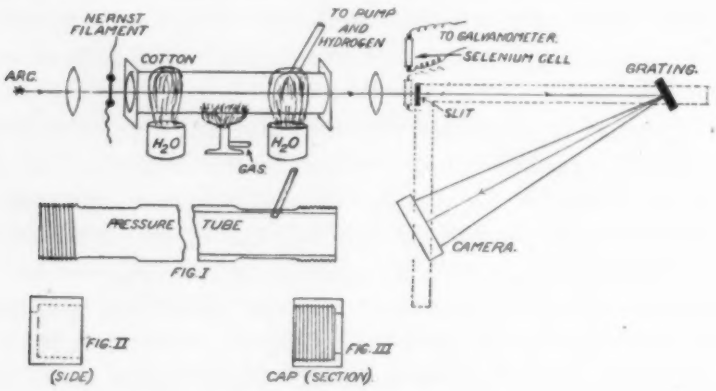
2. To determine the influence of variation in pressure upon the fluted absorption spectra.

[1] Phil. Mag. (6), 12, 1906.

[2] Astrophys. Jl., XXVI., No. 1, July, 1907.

METHOD OF INVESTIGATION AND RESULTS.

Apparatus.—In brief the apparatus consisted of a piece of seamless steel tubing $1\frac{1}{4}$ inches in diameter, 18 inches in length and wall $\frac{1}{16}$ inch in thickness, closed with glass plates hermetically sealed with sealing wax. About an inch from one end of this tube a hole was drilled into which was brazed a piece of $\frac{1}{4}$-inch brass tubing, through which the tube was exhausted. A small pan was placed in the central portion of the tube in which the sodium was confined. This kept the molten metal from running to the ends. This tube was employed for all of the work up to atmospheric pressure. For pressure higher, another one was constructed of the same type, except that the walls were much thicker, and the glass windows held on by threaded caps. It is shown in Fig. (1) of the diagram.



Figs. 1, 2, 3.

The internal diameter is $1\frac{1}{8}$ inches, thickness $\frac{1}{8}$ of an inch. Each end was threaded to receive the caps, which held the glass windows. This tube similar to one used by Wood.[1]

Just inside of the threaded portion on each end was turned down in a lathe to a thickness of $\frac{1}{32}$ of an inch, around which were wrapped the cotton pads. These were necessarily larger than those used on the low pressure tube, on account of the high temperature to which the tube must be raised to obtain the fluted effect at the higher pressures. This left 6 inches of the middle for heating area which

[1] Phil. Mag. (6), 1904.

would withstand the high pressure when the tube was at a bright red heat.

The caps for the ends were made of cast brass, turned out and threaded to fit rather tightly, a flange being left to hold the glass windows which were made with a special tool so that the fit would be close. They were taken from a piece of plate glass $\frac{3}{16}$ inch thick. Black sealing wax was used on account of its comparatively high melting point and hardness when cool.

As a source of light, the carbon arc was employed throughout, on account of its great brilliancy, care being exercised to keep the image of the positive pole on the slit.

In order to maintain a definite density of vapor it was necessary to keep the temperature of the tube constant. This was accomplished by using an ordinary Bunsen burner supplied with a modified type of fish tail tip. The electric furnace was first employed, but on account of lack of resistance, properly divided, a small enough increase or decrease of current could not be obtained. Besides, the furnace is very slow in reaching a steady state, requiring from one half to one hour at low temperature. It gives much more uniform layer of vapor, as the heating is on all sides. When great density is desired the use of Bunsen burners is more satisfactory, distillation to the colder parts of the tube taking place much less rapidly. To keep air currents from blowing the flame from the tube, and to give a more uniform temperature on all sides the flame was enclosed in an asbestos sheath, with a tin cap placed on the tube just above it.

A part of the work which did not require high dispersion was done on the 14-ft. concave grating. Later on the 21-ft. concave grating was employed, as the other proved unequal to the task of showing the small details of the spectrum.

The hydrogen used was produced by action of hydrochloric acid on zinc. Great trouble was at first experienced in high pressure work by the formation of fog in the tube, which made the atmosphere inside absolutely opaque for one or two hours. It was found however, that this fogging was the direct effect of impurities in the hydrogen. After elimination of these by passage through well-known solutions, little fog was observed.

METHODS FOR KEEPING CONSTANT DENSITY.

To determine when the density was at a certain stage, one of two methods was employed.

The first was used when taking a series of exposures in which the pressure was varied from 1 mm. to 760 mm. It was desirable to keep the actual absorption unchanged, noting the effect upon the flutings as a whole. Since the formation of flutings is due to the loss of only a few lines in the heads, it is natural to assume that, if the total absorption is unchanged, so is the density of the vapor, regardless of the change from bands into flutings due to variations in pressure of the gas. In order to determine the total absorption by the vapor, a selenium cell — kindly provided by Dr. Pfund — a Nernst filament and a sensitive galvanometer were used. In regulating the density of the vapor the light from the filament was passed through the tube and allowed to fall upon the selenium cell. The galvanometer deflection was noted; the filament and cell were removed, and the absorption spectrum photographed. If, now, this same density is desired for a different pressure of the gas, the filament and cell are replaced and the temperature of the tube varied until the galvanometer deflection is the same as before. In regulating the density for a certain state of absorption the Nernst filament was placed behind the tube so that when the galvanometer became steady, the filament and cell were removed, and the arc replaced. A photograph is taken, and for a change of pressure the above is repeated. A different deflection would mean simply a different density.

The increase of absorption upon increase of pressure of the foreign gas is due to the fact that the gas prevents the rapid diffusion of the metallic vapor, with the result that the partial pressure of the sodium is increased; in other words, more vapor is present in unit volume. It is therefore necessary to cool down the tube in order to reduce the absorption.

The second method for keeping the density constant was employed when working on the *D* lines. It consisted simply in focusing the *D* lines of the third order on a piece of ground glass, while the *D* lines of the second order were being photographed. Their widths were marked on the ground glass when the pressure was

1 mm., and it is assumed that this width of the absorption line marks a definite density of the vapor. Therefore, if a constant density is desired during any series of observations, all that is necessary is to maintain this width constant by slight changes in the temperature of the tube.

Absorption Spectrum under High Dispersion at 1 mm. Pressure.

The small pan was filled with metallic sodium and pushed to the middle of the tube, and the glass plate cemented to the ends. The tube was then exhausted and heated until the absorption lines appeared in the green and red. The *D* lines at this stage were about 1 Angström unit in width. The density was increased by gradually raising the temperature, and the absorption spectrum photographed at five different stages. The width of the *D*-line absorption region at the fifth stage was about 20 Angström units. The photographs show that the lines are of unequal intensity. In the first spectrum, which was taken with vapor of considerable density, certain groups of absorption lines have fused together, forming what appear to be broad bands with sharp edges. The sharpness and narrowness of the transmitted regions between the absorption lines is especially noticeable, the bright lines seeming quite as narrow as the iron lines. The general appearance is indeed quite similar to that of an emission spectrum. There is, even in this case, a slight suggestion of a grouping of the lines into bands. As we shall see later on, the admixture of hydrogen or some other inert gas brings out the fluted bands very distinctly by altering the relative intensity of the absorption lines. Figs. 1 and 2, Plate I., represent the green of the second order and Fig. 1 the red in the first order enlarged six times. Broken down under such high resolving power they are found to consist of an immense number of fine lines, extending from λ 6500 to λ 4500. They appear very sharp in the blue and blue-green regions, but as we approach the greenish-yellow they become less sharp, assuming a rather hazy form when λ 5150 is crossed. Although the dispersion is quite sufficient for studying individual lines, some of them still present a very complex form. This is seen in Fig. 4, Plate I., which also represents a portion of the orig-

inal negative enlarged nearly six times. The groups at λ 4615.5, 4624.5 and many appear as if composed of several narrower ones superimposed.

The red of the second order could be studied only visually owing to the mounting of the grating. It presented an even more beautiful appearance than the green and blue. The broad dark lines seen with lower dispersion are easily resolved, many of them being composed of from ten to fifteen very fine hair-like lines. These are spaced in irregular groups, one or two broader ones being placed in between. They are much finer than those in the green and are in much closer proximity. These were at their best when the *D* lines were just fusing together.

This fine line absorption spectrum of sodium vapor is not confined to the red and green, but immediately on each side of the absorbed *D*-line region the lines show nearly as fine and numerous as in the red. A rough estimate of the total number of absorption lines given by sodium vapor, at low pressures, is about 8000 or 10,000. No doubt higher dispersion will reveal many more.

Although the red could not be photographed in the second order on account of the limit to the movement of the 21-ft. grating, a good idea could be gotten from photographs taken in the first order. They appear very much as do those in the green.

Wratten and Wainwright plates (panchromatic) were used and found very satisfactory in this region.

EFFECT OF INTRODUCING HYDROGEN.

1. The appearance of the flutings as a whole as the pressure is increased from 1 mm. to 11 atmospheres.

2. The effect upon individual lines and groups of lines.

FLUTING WITH PRESSURE 1 MM. TO 760 MM.

In studying the appearance of the flutings as the pressure of the gas changes, a series of photographs were made varying the pressure from 1 mm. to 760 mm., by seven steps, keeping the actual degree of absorption the same. This was accomplished by the method already described with the selenium cell and sensitive galvanometer. Two sets of photographs were taken, each with a

definite density of vapor. The principal set was with a density such that the flutings were just entering their best stage, and when the central ones in the green were partially absorbed. All show the same thing, viz., the extent to which the bands become fluted varies directly with the pressure up to atmospheric, which seems to be a critical value; as the appearance remains practically the same, for an increase of pressure amounting to several atmospheres.

The photographs at 1 mm. pressure, taken in the first order with the 14-ft. grating, presents a series of irregularly spaced fine lines from λ 4600 to λ 5200, from which point they appear more regular, breaking up into a group of three small bands between λ 5100 and λ 5136, six even more regular ones between λ 5180 and λ 5220, and three others between λ 5240 and λ 5270. All of these lines, except those included in the three named sets of bands, show a tendency toward fluting, as soon as the pressure is increased above a few centimeters, the fine lines composing the heads of the flutings dropping out or becoming diminished in intensity. Points which at 1 mm. show as absorption lines gradually become replaced by transmitted regions, smoothing out the irregularities, causing the spectrum as a whole to acquire a uniform fluted appearance.

The three sets of small bands between λ 5200 and λ 5300 simply grow more feeble, until at 60 cm. pressure they fade almost completely, the spectrum becoming nearly continuous. The series of fine lines between λ 4700 and λ 4800 not only flute regularly, but begin to fade away, leaving no trace of lines below λ 4730.

The second series of photographs with greater absorption shows the same general effect, except that the extreme ends of the green absorption region between λ 4700 and λ 4800; and between λ 5100 and λ 5320, come out more strongly owing to the greater absorption. The group at λ 5200 was still visible at 760 mm. This was also true on the violet side, *i. e.*, the fine line flutings pushing out further into the shorter wave-lengths. The central large flutings between λ 4850 and λ 4950 become much weaker, as the pressure is increased. In order therefore to photograph them and the more intense bands at the same time a mechancial screen was employed. This was placed in front of the camera box, and operated by a small motor. It consists of a piece of cardboard with a section cut out about the

length of the portion of the spectrum needing more exposure. This, when moved back and forward in front of the plate shades in the brighter portion of the field into the darker. Absorbing screens were tried but with little success. All possess too much general absorption and further, no combination could be found that would give a narrow transmitted band in the portion of the spectrum desired.

In the red at the lowest pressure there appears between λ 6000, and λ 6100, beginning indistinctly at the former, a series of eight bands or semi-flutings, which aside from becoming a little more distinct, undergo little change as the pressure is increased. These correspond to the invariable bands observed in the greenish-yellow region. Reaching atmospheric pressure there are eight much longer ones which appear between λ 6100 and λ 6500; being, however, more sharply defined.

Close examination of the photograph shows a slight lack of definition at 76 cm. compared to that at 2 mm. This will be discussed under head of "Effect upon individual Lines," but it is mentioned here as a forerunner to the discussion of the fluting under high pressure.

EFFECT OF VARIATIONS IN PRESSURE UPON FLUTING. ONE TO TEN ATMOSPHERES.

Series of exposures were made with the pressure varying from one to ten atmospheres, increasing from one and one-half to two atmospheres each time. The result is shown in Fig. 2, Plate II. No special trouble was experienced in getting the spectrum to flute up to three atmospheres. But above this it required two good Bunsen burners, with strong blast, well inclosed, to bring the flutings out distinctly. This made the tube a bright cherry-red color, and after reaching 10 atmospheres, fear of breakdown limited a further increase of pressure.

There was a very faintly fluted spectrum obtained at the last named pressure, but in the process of printing this is lost, and therefore is not shown here, but it affords a good indication of what might be expected at higher pressures.

The spectrum at atmospheric pressure as was mentioned above

shows a lack of sharpness. This is also true at higher pressures. At nine atmospheres there remains only a well fluted continuous spectrum. This is perfectly distinct at this pressure, although at ten atmospheres scarcely any trace of it appears.

The behavior in the red was observed visually, under the same pressures and similar changes noticed, except that the flutings and lines disappear at much lower pressures than do those in the green. Owing to the fact that the vapor was made under such high pressure and consequently so dense, very little light could be gotten through the tube above five atmospheres, and photographs could not be obtained in the second order. Exposures in the first required from three fourths to one and one half hours, and the red region, even with a comparatively wide slit could scarcely be seen, only presenting a weak continuous appearance. At ten atmospheres the absorption is very general in its nature, practically all of the red-yellow and blue-green is gone, leaving the violet much stronger than the rest.

Effect of Pressure upon Individual Lines and Groups of Lines.

In this portion of the investigation the second order of the 21-ft. grating was used where possible.

The tube being cleaned out and a lump of sodium inserted, it was exhausted to a pressure of 1 mm. An exposure was made when the absorption lines were most numerous and the definition greatest. Then hydrogen was admitted to atmospheric pressure, the tube being left connected to the gas supply. This was done to make sure that the pressure in the tube did not rise on account of the property possessed by metallic sodium when heated, to occlude great quantities of hydrogen.

A photograph was taken when the conditions of absorption were as near as possible the same as at the lower pressure. The result is shown in Figs. 1 and 2 of Plate I., Fig. 4 being given to show the width of the absorbed region at the *D* lines, at density used. The heads of the flutings can now be measured accurately and are given below. With lower dispersion they appear as "single line" heads, and were measured as such by Moore, but, as can be seen from the

data, they consist of one, two and sometimes three lines, some of them having no definite head. The form of these heads is given along with that produced at 760 mm., so that some idea can be gotten of the phenomena taking place.

The lack of definition mentioned in the preceding part is seen to be much more pronounced in the green than in the red, especially at the longer wave-lengths. Instead there is more actual absorption at 760 mm., many of the strongest bright lines being cut out completely, and feebler ones making their appearance. These remain sharp compared with the yellow, green and blue absorption lines near λ 5700 and λ 4600 respectively. The intermediate regions between λ 4650 and λ 4920 show only the fluting effect. The two spectra have nearly the same intensity at this point, but at λ 4600 comparatively little light is absorbed.

APPEARANCE OF HEADS OF FLUTINGS IN GREEN. WAVE-LENGTHS ACCURATE TO .01 ANGSTRÖM UNIT.

	Vacuum of 1 mm. p.	Atmospheric Pressure.
1.	4766.71 66.97 doublet. Slightly broken effect at 4782.92 and 4792.95	Fuses together, no shift.
2.	4819.75 heavy single line. Very irregular fluting.	Spreads some and shifts to 4819.85
3.	4837.15 37.31 doublet.	1 Shifts. 4837.26 2 Does not shift. 4837.31
4.	4865.15 very heavy. 4865.30 weak. Consists of a series of irregular lines.	Breaks into triplet. 4865.04 65.30 65.50 Breaks into series small doublets increasing in size to head.
5.	4894.48 heavy single.	Shifts to 4894.37
5.	Semi break at 4911.40	Breaks into symmetrical doublet. Resembling Zeeman effect. 4911.84 and 20.1 The whole fluting consists of irregular groups of doublets and triplets.
6.	4932.41 .54 doublet.	Fuse into cone at 4932.38

7.	No definite lines.	Transformed into hazy triplet at 4962.04 .23 .47
		Lines irregular all through
8.	5000.94 heavy. 1.12 weak.	Forms hazy doublet 4000.86 5001.10
		1 shifts. 2 widens toward violet.
		Very irregular series shown in Fig. 2, Plate III.
9.	5039.92 40.08 triplet. .18	Contracts to close doublet almost inseparable at 5039.98
10.	Is weak and hard to notice 5079.65 5076.89 doublet.	Remains doublet, both shifting symmetrically to 5079.56 5079.89

There are besides these the seven small bands, or groups of lines which produce a sort of "rolling" effect, and appear like this:

These are found in the blue between λ 4703.46 and λ 4810.16. They are not apparent at vacuum, but are brought out at 76 cm. The wave-lengths are given below:

4703.46	4741.59
4704.78	4767.22
4716.10	4810.11
4727.89	

The bands found in the yellow green, and unaffected at atmospheric pressure are given below:

5100.5	5184.1	5240.1
5112.6	5196.2	5252.1
5124.7	5208.3	5264.2
5136.8	5223.5	5276.3

These bands in the violet are clearly noticeable, at eleven atmospheres, while those in the yellow green stand fairly well up to six atmospheres.

Flutings in the Red and Orange.	
5997.82	
6019.23	
6042.05	Well prounced at vacuum, only slightly brought out at pressure.
6080.40	
6099.64	
6119.12	
6141.01	
6168.00	

6179.23	Slightly visible at vacuum but brought out clearly by pressure.
6185.06	
6219.87	
6276.29	
6329.44	
6373.88	
6417.50	
6464.62	
6512.36	Not at all visible at vacuum, but strongly fluted by pressure.
6559.64	
6610.30	

In the orange the two spectra are about equal intensity, and the lines are fused together, but as the pressure is raised to 760 mm. the intensity is much less, and the fluting is more pronounced.

One of the chief characteristics of the absorption spectrum is the doublet formation observed at atmospheric pressure. A small part of this can be seen very clearly in Fig. 3, Plate I., between λ 5001 and λ 5025, which represents a portion of the original negative enlarged about two and one half times. In the red part of the spectrum the reverse is true, that is, this doublet formation is more evident at 1 mm. pressure, than at 760 mm. Between wave-lengths λ 5900 and λ 6200 this is very common, especially near λ 6100.

This characteristic of doubling by the lines extends up to 6600 and appears only at very low pressures, disappearing at atmospheric pressure.

In the enlarged portions represented by Figs. 1 and 2, Plate I., Fig. 3 shows at 760 mm. absolutely no regularity, whereas at 1 mm. the groups between λ 5001 and λ 5025, break up or are transformed into the regular series shown.

Many of the lines are displaced as much as .15 A.U. for a change of 760 mm. One line out of a group will move to the red, the others remaining in this same position.

Nearly all of those which shift at all none are in the direction of longer wave-length, although many are displaced toward the violet. This apparent change of position does not differ in amount between wave-lengths λ 4600 and λ 5100, the average being about .11 of an Angström unit. At λ 5006.2 can be plainly seen the difference in the absorption at 760 and 2 mm. respectively.

Judging from the green absorption region one would immediately look for similar changes in the red. But, in fact, although all of

the lines were examined, there can be noticed only a few instances of unsymmetrical broadening or apparent shifts.

In view of the fact that so many changes occur in the absorption spectrum of sodium vapor, in the presence of hydrogen as the pressure is changed from 1 mm. to 760 mm., some sort of chemical combination seems altogether possible. Consequently to decide this point the hydrogen was replaced by nitrogen. A fresh piece of sodium was used and the tube cleaned before taking a photograph. The tube while heated was exhausted; air was allowed to enter the tube, was again exhausted, etc., several times, each time allowing air to enter the tube at atmospheric pressure. On account of the great affinity of sodium for oxygen little else remained in the tube except nitrogen. A photograph was obtained under the same conditions as with hydrogen, and the two spectra showed exactly the same results line for line.

Study of the D Lines.

The effect of pressure upon the *D* lines is much different from that on the other absorption lines of sodium. Photographs were taken with pressure varying from 1 mm. to 12 atmospheres, the density being kept constant by the method already described. Except for a slight decrease in sharpness no change in the spectra was observed as the pressure was changed from 1 mm. to 760 mm. This can be seen only when one of the *D* lines is about 2 A.U. in width. Any greater density causes the absorbed space to become so broad and hazy that this effect can not be detected, and with very low density, even at the highest pressures, the *D* lines should remain very fine and sharp.

Fig. 3, Plate II., shows the effect of high pressure. There seems to be an unsymmetrical broadening, which extends to the longer wave-length. It can be best described in this way. If we heat up the tube at 760 mm. pressure until *D* is about 2 Angström units in width, then remove the burner, and allow the tube to cool, the two *D* lines of course will be seen to die away symmetrically, getting narrower until they disappear. If, now, we raise the pressure up to 10 atmospheres, and again observe the decay of the lines, the fine *D*'s will be seen just on the edge of the shadow which vanishes

some time before the *D* lines themselves. It is much easier to observe this change than to photograph it, but an excellent idea can be gotten from the figure on Plate II.

This effect is very much the same as that observed by Julius in working with anomalous dispersion of sodium vapor, where he causes unsymmetrical broadening of the *D* lines, either one or both, by merely changing locality of heating. The source of heat was obtained by an electric current passing through two nickel wires running parallel through the tubes containing the vapor.

The heating in the present instance was a small flame under the tube. By using an electric the point could be immediately decided. But for lack of time the author would have tried this.

SUMMARY.

The results of the investigation may be summed up as follows:

1. Under high dispersion the absorption spectrum of sodium vapor in the region investigated is found to consist of approximately 10,000 lines, the region of absorption extending mainly from λ 4500 to λ 6700, which was as far as results were obtained.

2. The bands can be divided into two classes: first, those in which the extent of fluting varies directly as the pressure is increased from 1 mm. to 760 mm., and second those which are not affected much by pressure.

3. As the pressure is increased to 8 atmospheres, the absorption spectrum becomes continuous in appearance, the fluting disappearing almost completely at 10 atmospheres.

4. The transformations of the lines which occur as the pressure is raised from 1 mm. to 760 mm. may be briefly stated as follows:

(*a*) Many of the lines between λ 4600 and λ 5100 shift their positions as much as .15 A.U. caused either by unsymmetrical broadening or by the pressure of the foreign gas.

No such effect is observed in the red part of the spectrum.

(*b*) A single line is changed into a doublet, and a doublet into a triplet, although the reverse is more often true.

(*c*) The whole spectrum is characterized more or less by a doublet formation, which is more noticeable in the shorter wave lengths at 760 mm., and in the longer wave-lengths at low pressures.

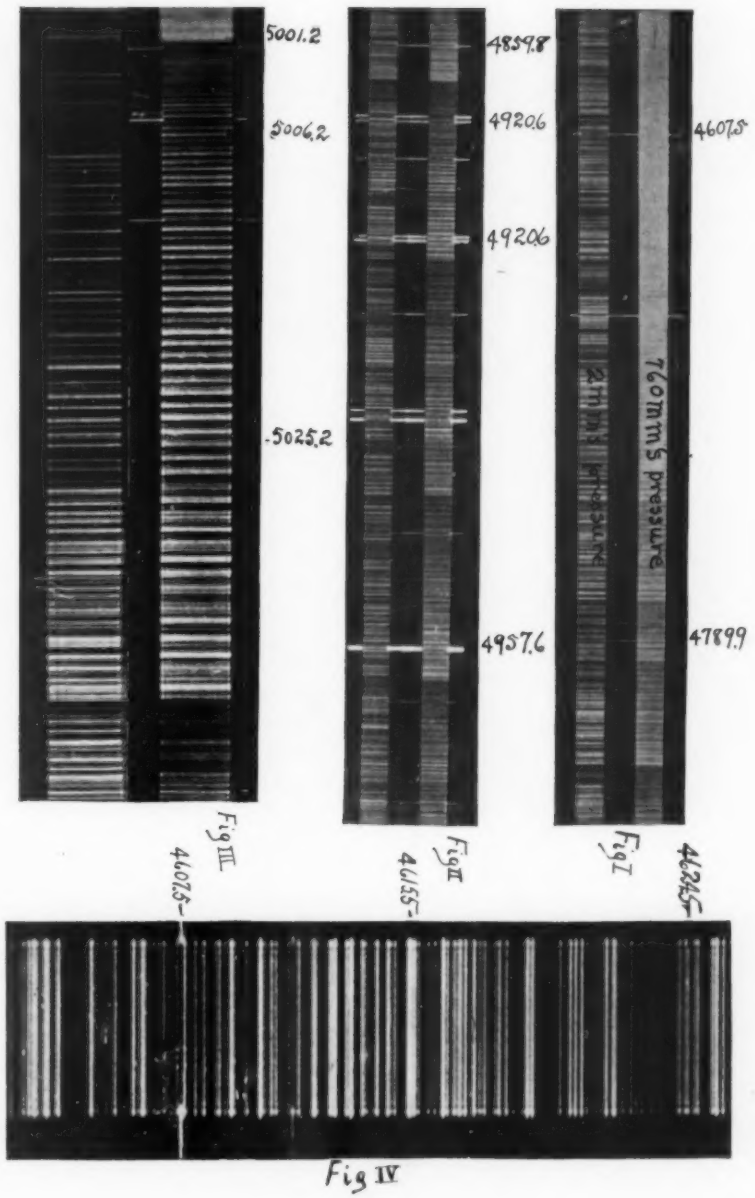
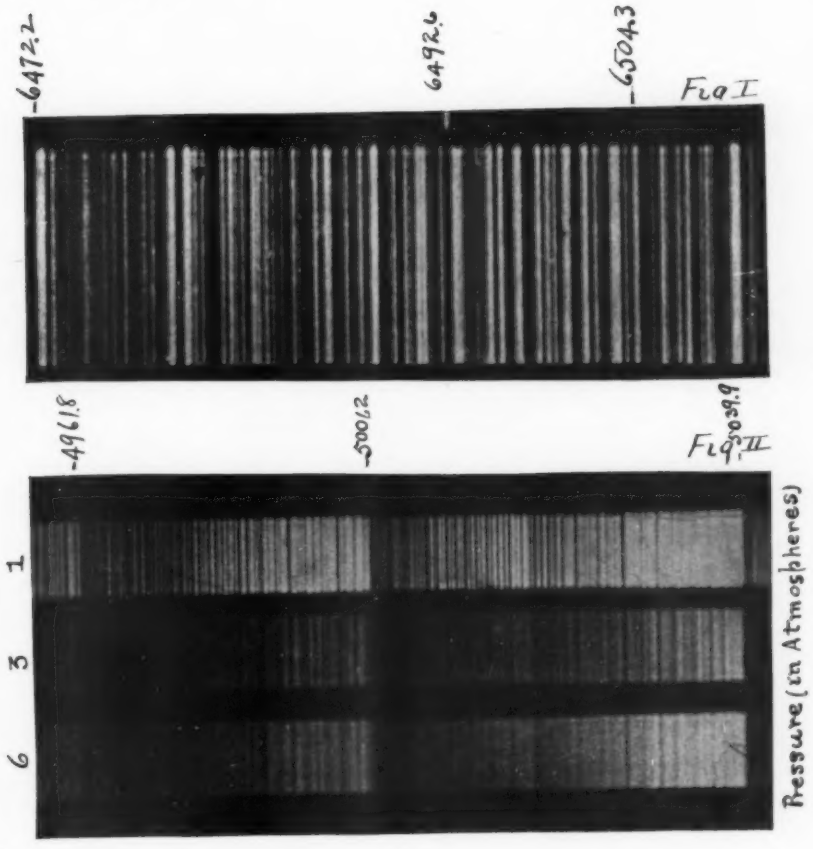


PLATE I. CLINKSCALES.

The Absorption of Sodium Vapor.

To face page 608



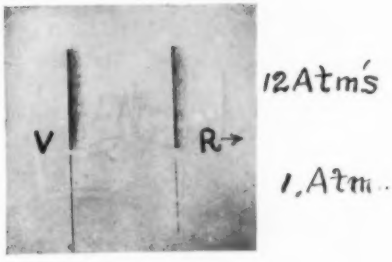


Fig III

PLATE II. CLINKSCALES.

The Absorption of Sodium Vapor.

(*d*) The *D* lines, when *D* is about one Angström unit in width, are unsymmetrically broadened nearly 5 A.U. under 12 atmospheres pressure, but at extremely low pressure, remains very sharp and shows no broadening.

5. No difference can be noticed in the absorption spectra for the different gases used.

In conclusion I wish to acknowledge my great indebtedness to Professor Wood, who suggested the work and under whose direction it was carried out.

JOHNS HOPKINS UNIVERSITY,
June, 1908.

THE ELASTIC PROPERTIES OF PLATINUM-IRIDIUM WIRES.[1]

BY K. E. GUTHE AND L. P. SIEG.

1. About two years ago one of the authors[2] found that platinum-iridium wires show a remarkable deviation from the elastic behavior of ordinary wires when used as suspensions for experiments on torsional vibrations. This peculiarity was especially pronounced in a wire containing 40 per cent. of iridium.. It was shown that for the amplitudes employed — ten degrees and less — the torsional period of cylinders suspended by this wire decreased proportionally to the amplitude, the decrease amounting to nearly two per cent.; and further, that the exceptionally large logarithmic decrement also is, for small amplitudes, a linear function of the amplitude. With different moments of inertia the logarithmic decrement remained unchanged and the various curves, obtained by plotting periods and amplitudes coincided, after proper scale reduction was made. In a wire containing 25 per cent. of iridium these peculiarities were less pronounced, but still noticeable.

The present investigation was undertaken in order to extend the observations to larger amplitudes and to study the influence of different percentages of iridium upon the elastic properties of platinum-iridium wires. We are under obligation to Dr. Heraeus, of Hanau, for preparing the alloys and drawing the wires needed in our experiments.

2. *Apparatus.*—The ends of the wire were soldered in brass cylinders one of which served as the supporting rod and was clamped in a wall bracket. The lower cylinder formed a part of the vibrating system; it was accurately turned to a diameter of about 6 mm.,

[1] This investigation was carried out with the aid of a grant from the Elizabeth Thompson Science Fund.

[2] K. E. Guthe, Proc. Iowa Ac. Sci., 15, p. 147, 1908; abstract in PHYS. REV., 26, p. 201, 1908.

leaving at its lower end a flat disk of about 9 mm. diameter thus forming a support for a number of hollow cylinders which fitted snugly over the supporting pin. By the use of different hollow cylinders the moment of inertia of the moving system could be varied from 366 g.-cm^2 to 1344.5 g.-cm^2. At its lower extremity the pin carried two small mirrors with their backs glued together. An optical test showed that they were parallel to within a few seconds.

The wire was suspended with the axis of the vibrating cylinder in the center of a semicircular scale of 149.2 cm. diameter. The mirror reflected upon the scale the image of a hole with a fine crosswire placed in front of an arc light. A lens between mirror and crosswire focused the image of the latter upon the scale. An angular displacement of one degree of the mirror produced thus a displacement of 2.6 cm. of the image. This arrangement of the apparatus allowed to read amplitudes of the vibrating system from 765 to 675, 405 to 315 and 90 to 0 degrees. In later experiments the half circle was replaced by a complete circle so that practically the whole series of amplitudes could be observed.

3. *Method.*—The passage of the image of the crosswire through the zero point was recorded by an observer upon a chronograph which latter was also provided with means of marking double seconds from an astronomical chronometer. The intervals between successive coincidences were calculated from the chronograph record. When the amplitude had died down to a few degrees the eye and ear method was used for the determination of coincidences and the calculation of the average half periods.

It was impossible to obtain by this subjective method a very accurate determination of the time between two successive passages of the crosswire through the zero. It was therefore necessary to calculate the period from longer intervals, usually, except for the first 20 swings, from 40 vibrations, each corresponding to a half period. In the following tables and figures the values given under "period" refer without exception to half periods, expressed in seconds. The curve for the period when plotted as a function of the time is for large amplitudes nearly parallel to the X axis, but soon bends downward and forms a curve convex towards the origin. The periods calculated in the manner described will therefore, for

the greater part of the curve, be slightly larger than the actual periods. By taking different intervals, for example, of 20, 30 and 40 vibrations we assured ourselves that the error thus introduced did not exceed the error introduced by our subjective method of recording the passage of the crosswire, which is about one one-thousandth of a second when the period is calculated from 40 swings. The accuracy of the values for the first 20 periods is necessarily smaller since no such large intervals could be chosen for their calculation.

The amplitudes were plotted on cross-section paper and, unless otherwise stated, the amplitudes given in the tables are the values read off from this curve as corresponding to a given average period.

4. *Period and Amplitude.* — We began our experiments with the finest 40 per cent. platinum-iridium wire received from Heraeus. Its length was 80.4 cm., and its diameter 0.194 mm. Since former experiments had shown that previous heat treatment greatly influences the elastic properties, and since we wished to have all wires in the same condition, we annealed the wire under the load which it was to carry, by sending a current of 2.0 amperes through it. This raised its temperature to a bright yellow heat and straightened it. Even at these high temperatures it can still support a considerable load.

Our observations corroborate fully the results obtained in the earlier experiments, the increase in period, for small amplitudes, being proportional to the amplitude. However, with larger amplitudes, the rate of increase becomes smaller and finally the period approaches a constant value, though even with amplitudes of 700 degrees there is still a slight increase. A large number of experiments were made, varying such conditions as load, moments of inertia and temperature. In all cases the curves show the same characteristic form. The data under A in Table I., plotted in Fig. 1, are taken from one of these experiments. The first column contains the number of half swings, the second the amplitude in degrees, the third the half period in seconds. It is seen that starting with an amplitude of 767 degrees the decrease of the period reaches the enormous value of 7 per cent., of which six sevenths fall upon the interval from 250 to 0 degrees. For the last 60 degrees the decrease is proportional to the amplitude.

5. *Change of Length.* — We next shortened the wire to three quarters of its original length. This was done for the double purpose of testing if the peculiarities might possibly be due to some lack of homogeneity in the wire, and of obtaining still larger torsional strains.

TABLE I.

40 *Per Cent. Platinum-iridium Wire No.* 2.

A. Jan. 30; *Length* 804 *mm.; Moment of Inertia* 366 *g.-cm*^{2}; *Temp.* 14°.3 *C.*
B. Apr. 19; *Length* 602 *mm.; Moment of Inertia* 838.5; *Temp.* 21°.0 *C.*
C. May 7; *Length* 403 *mm. Moment of Inertia* 1344.5; *Temp.* 21°.5 *C.*

No. of Swings.	*A*		*B*		*C*	
	Amplitude.	Period.	Amplitude.	Period.	Amplitude.	Period.
0	767	4.788	786	6.435	765	6.459
2	700	.783	732	.432	725.4	.458
4	640	.778	687	.427	692	.456
6	565	.770	649	.423	651	.455
8	506	.762	614	.420	630	.453
10	443	.753	580	.417	597	.452
12	388	.744	548	.413	570	.451
14	340	.731	516	.409	544	.450
16	297	.715	486	.405	517	.449
18	260	.695	454	.402	494	.447
20	228	.673	426	.396	470	.445
25	159	.616	358	.385	421.4	.440
30	116	.573	298	.370	372	.435
35	84	.543	243	.353	326	.429
40	65	.521	197	.334	284	.422
45	52	.503	157	.308	246	.414
50	44	.487	122	.281	212	.405
60	32.6	.466	74	.226	152	.385
70	25.3	.455	47	.174	103	.351
80	20.2	.447	32	.134	67	.316
90	16.6	.440	24	.105	46	.278
100	13.8	.434	19	.086	31.4	.244
120	10.1	.428	12	.062	17.1	.193
140	7.4	.426	8	.036	10.4	.161
160	5.8	.424			6.9	.140

The results are given in Table I. under B and plotted in Fig. 1. The period closely corresponds to what would be expected for an ordinary wire, considering the change in length and torsional moment. The conclusion is therefore justified that the wire had a uniform structure throughout its length. As we expected, the

period did not drop nearly as fast at the beginning as it did with the longer wire. A decided falling off in period did not occur until the amplitude had decreased to about 500 degrees. The final decrease of period is much more rapid than with the longer wire.

When the wire was shortened to 403 mm., half the original length, the influence of the length upon the variation of the period becomes still more marked. For the largest amplitudes the period is practically constant, and the final drop very rapid. The results are

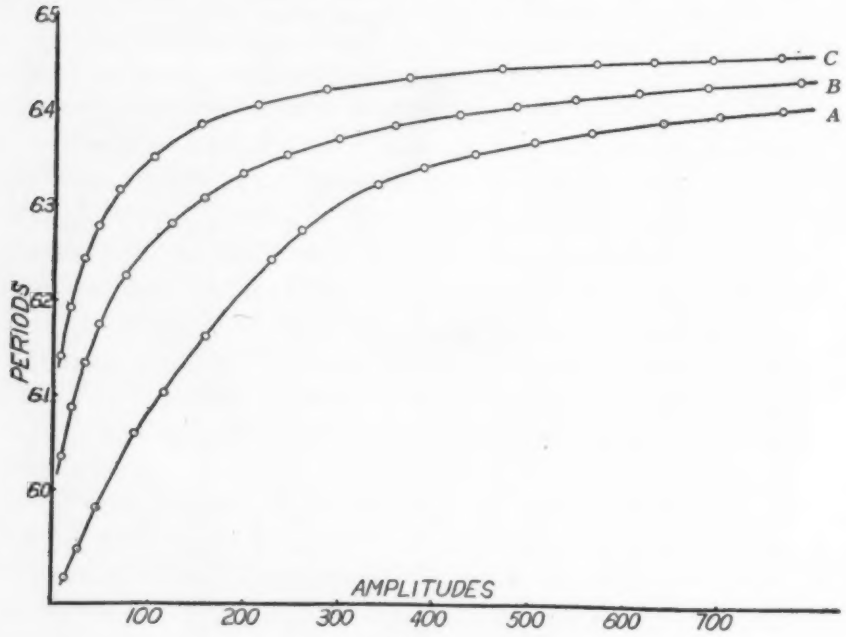


Fig. 1.

given under C in Fig. 1 and Table I. For comparison the scale for the period of A in the figure has been chosen one and one third times larger than that for B and C.

6. *Change of Moment of Inertia.*—Varying the moment of inertia of the moving system did not change the character of the curves. It became, however, soon apparent that the phenomenon was greatly influenced by the conditions under which the vibrations were produced. We could not in general obtain Guthe's results with small amplitudes, namely, that all curves coincide when plotted on

corresponding scales. Only in exceptional cases when the conditions of temperature and initial amplitude must have been exactly alike, a close agreement between the curves was found. Table II. is an example of such a case. Only the amplitudes and periods are given. The scale for the periods in the experiment of March 6 was reduced to that of March 13 by multiplying the periods by 1.692.

TABLE II.

40 per cent. Platinum-iridium Wire No. 2, Length 804 mm.
A. March 13 ; Moment of Inertia 1344.5 g.-cm² ; Temp.= 21°.1 C.
B. March 6 ; Moment of Inertia 473.7 g.-cm² ; Temp.= 23°.0 C.

No. of Swings.	A		B		
	Amplitude.	Period.	Amplitude.	Period.	Reduced Period.
0	742	9.267	730	5.480	9.273
2	688	.264	670	.477	.267
4	635	.260	623	.475	.264
6	585	.254	572	.471	.257
8	540	.245	523	.466	.249
10	492	.235	474	.459	.237
12	453	.225	423	.451	.223
14	415	.214	379	.440	.205
16	380	.198	343	.427	.182
18	346	.182	308	.415	.162
20	313	.165	277	.403	.142
25	241	.124	209	.374	.093
30	187	.082	158	.346	.046
35	143	.037	118	.320	.001
40	110	8.994	90	.295	8.960
45	83	.953	72	.273	.921
50	68	.915	57	.253	.888
60	42	.860	38	.224	.839
70	32	.822	28	.205	.806
80	24	.798	21	.193	.786
90	19.3	.783	15	.185	.773
100	15.3	.768	13	.177	.759
120	10.1	.746	8	.161	.732

In most cases such close agreement could not be obtained. The slope of the curve does not alone depend upon the initial amplitude but also upon whether or not the wire had been kept for some time in a twisted condition before being released or had been kept swinging continuously before the experiment was started.

7. *Influence of the Initial Amplitude.*—For this part of our investigation we made use of the old specimen of platinum-iridium wire which had not been vibrated for considerable time. This had been obtained from Heraeus some years ago; it was marked 40 per cent. platinum-iridium, had a length of 402 mm. and a diameter of 0.192 mm., *i. e.*, the same diameter as the finest wire, more recently secured.

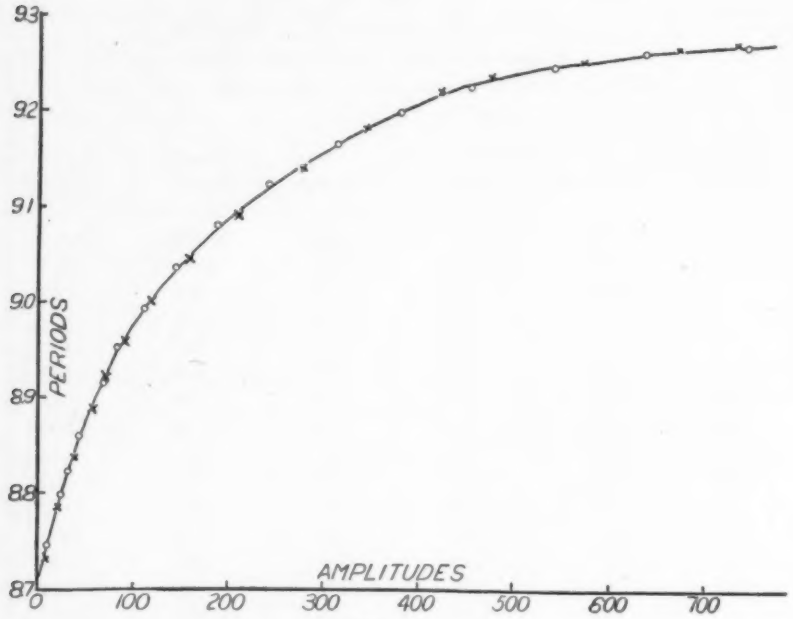


Fig. 2.

The wire was heated under a load which it was to carry, 240 g., the moment of inertia of the suspended system being 979.5 g.-cm^2. After being placed in position the wire was allowed to rest for some days. The first set of readings was taken with a small initial amplitude, 21 degrees; after which the wire was allowed to rest a day. Then the initial amplitude was increased to 47.3 degrees, and again a day's rest given; this process was continued, the amplitude being increased each day. The results are found in Tables III.–VIII., where not alone the periods and corresponding amplitudes are given (in columns 4 and 5) but also the observed amplitudes and the observed times of passage of the image of the crosswire through

TABLE III.

40 Per Cent. Platinum-iridium Wire No. 1; Length 40.2 cm. Maximum Amplitude 21 Degrees; Temperature 22°.3 C.

No.	Amplitude.	Coincidences.	Period.	Corr. Ampl.	Log. Dec.
0	14.3	0′ 22″.66			
10	13.5	1 14 .81			.00227
20	12.85	2 6 .97	5.214	12.77	
30	12.05	2 59 .01			222
40	11.6	3 51 .20	5.212	11.6	
50	10.95	4 43 .25			216
60	10.5	5 35 .37	5.213	10.7	
70	10.2	6 27 .53			194
80	9.6	7 19 .62	5.211	9.6	
90	9.2	8 11 .76			189
100	8.8	9 4 .00	5.213	8.8	
110	8.5	9 56 .00			175
120	8.12	10 48 .10	5.210	8.12	
130	7.85	11 40 .12			167
140	7.52	12 32 .38	5.207	7.55	
150	7.25	13 24 .39			180
160	6.92	14 16 .45	5.209	6.92	
170	6.60	15 8 .61			136
180	6.50	16 0 .54	5.206	6.50	
190	6.30	16 52 .78			120
200	6.15	17 44 .85	5.205	6.15	
210	6.00	18 36 .88			90
220	5.90	19 28 .82	5.204	6.00	
230	5.85	20 20 .94			75
240	5.70	21 12 .85		5.55	

the zero point (under coincidences in column 3). These tables show the very interesting fact that the wire makes a larger number of vibrations when started with a small amplitude than when started with a larger amplitude. This is brought out clearly in Fig. 3, in which the zero points from which the number of swings are counted are placed the farther towards the right the larger the initial amplitude.

In each successive series the wire makes about 50 vibrations less in reaching an amplitude of five degrees. Thus it takes only 125 half vibrations to fall from 364.3 degrees to 5 degrees while in the first case 200 swings are required to cover the interval from 14.3 to 6.15 degrees. If we had chosen a still smaller amplitude than five degrees for our comparison the difference would be still greater, judging from the slope of the curves at the small amplitudes.

All six curves may be made to coincide if the scale for the vibration number is reduced. This is shown in Fig. 4. The values of the amplitudes from Table VIII. are plotted as ordinates, the number of half vibrations as abscissæ. Then for the maximum recorded amplitude in each set of observations the corresponding position in the diagram was found, and from this point on the amplitudes were plotted by reducing the scale of the number of vibrations of each set, multiplying by a constant reduction factor. These factors

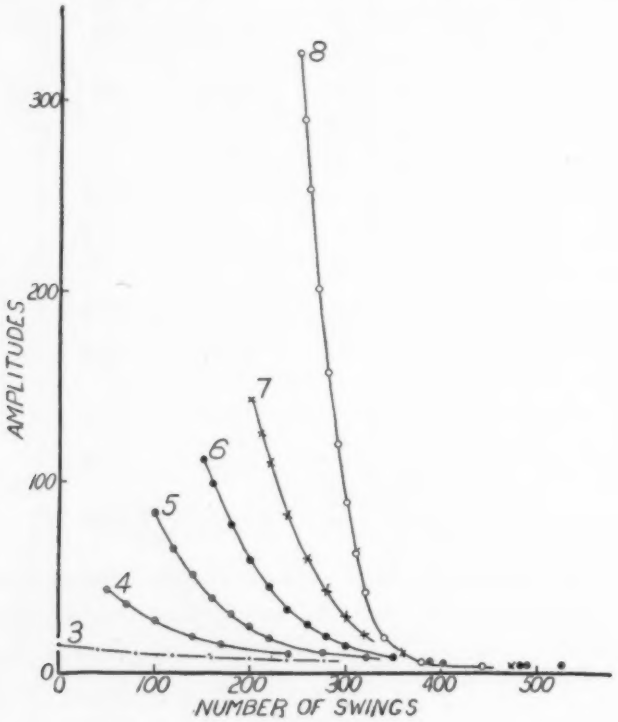


Fig. 3.

are 9.3/20, 7.225/20, 6.15/20, 4.75/20 and 2.915/20 respectively for the series beginning with amplitudes of 117, 86.4, 47 and 22.3 degrees.

It seems to be a curious coincidence that the reduction factor in the first case, namely, 9.3/20 = 2.15 is almost exactly the ratio of the two maximum amplitudes, 364.3/168.9 = 2.16. In all other cases the reduction factors are smaller than the ratio of the maxi-

mum amplitudes, the difference increasing as we proceed towards the series with smallest amplitude.

As could be expected the curves for the period as functions of the amplitude is greatly dependent upon the initial amplitude, the change in the period being the smaller the smaller the initial amplitude. Fig. 5 shows this very clearly.

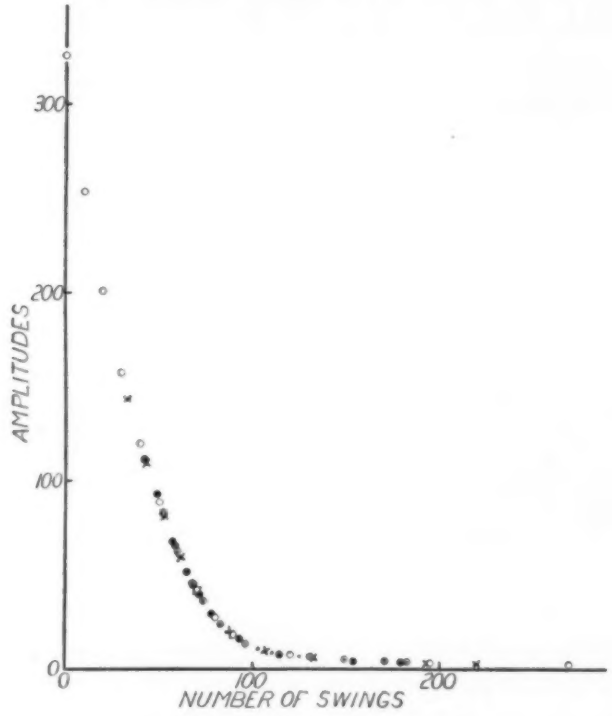


Fig. 4.

8. *The Logarithmic Decrement.*—Though the logarithmic decrement loses all significance in elastic phenomena of this kind, it gives a very interesting curve for the various cases considered. From the results discussed in the last paragraph it follows immediately that the logarithmic decrement will be the smaller the smaller the initial amplitude, though in all cases the mathematical expression for its dependence on the amplitude must be the same except for a numerical factor. When plotted as a function of the amplitude the decrement starts as a straight line for small amplitudes, but

reaches a maximum for an amplitude which is the farther towards the left the larger the initial amplitude; this maximum becoming most pronounced in short wires. Beyond this maximum the decrement decreases at first rapidly, then slower, and probably reaches a minimum for large amplitudes. In Fig. 6 the logarithmic decrement is plotted as a function of the amplitude for three series, (*a*) A in Table I., (*b*) C in Table I., and (*c*) Table VIII.

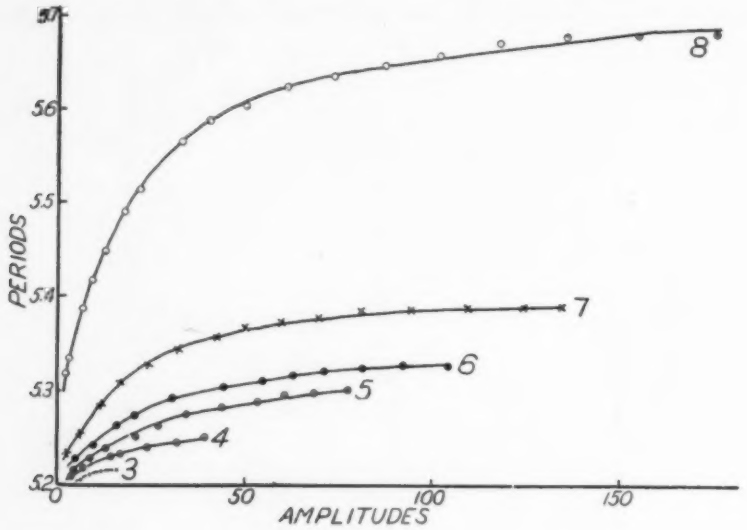


Fig. 5.

The curves show a great similarity to those of magnetic permeability of iron as a function of the magnetizing field intensity. Bouasse[1] who, for many years, has studied the elastic behavior of ordinary wires has obtained very similar curves to ours in the case of an iron wire. He also found a minimum for large amplitudes.

9. *Hooke's Law.*—The preceding results suggest that the wire may have been strained beyond its elastic limit or that the elastic after-effect may play an important rôle in the phenomenon. The resting point of the wire remained, however, remarkably constant during all of our experiments and previous investigation by one of us showed that, when tested by the ordinary method, the elastic after-effect in these wires is quite small, indeed, smaller than for

[1] Bouasse and Carrière, Ann. de Chim. et Phys., 14, p. 190, 1908.

steel. These tests were made by holding the suspended cylinder in a fixed position twisting the torsion head 360 degrees, holding it there for two or three minutes, turning the torsion head back and determining as soon as possible afterwards—about half a minute—the new resting point. No displacement greater than a degree was observed, the after-effect slowly disappearing in the usual manner.

We repeated an experiment mentioned in the earlier publication, namely, a test of Hooke's law for these wires. The wire was connected by a brass pin to a steel wire for which the period had pre-

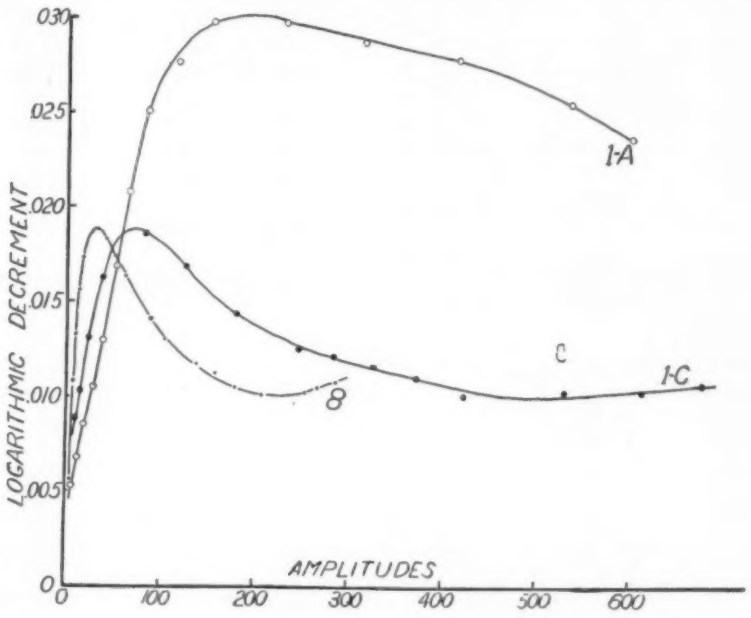


Fig. 6.

viously been determined and found constant. Both wires were soldered into the pin, which carried a mirror. The platinum-iridium wire formed the upper part of the system and was clamped in a torsion head; the steel wire carried at its lower end a load, a part of which was a thick brass plate fitting a groove in a holder, fastened to the floor. This prevented a twisting of the lower end of the system, but allowed the plate to slide up and down in the groove when the length of the wire changed due to the twist. The load

TABLE IV.

Same Wire as in Table III. Maximum Amplitude 47.3 Degrees; Temperature 24°.9 C.

No.	Amplitude.	Coincidences.	Period.	Corr. Ampl.	Log. Dec.
0	43.6	0′ 6″.51			
10	39.2	0 59 .03	5.250	39.0	.00458
20	35.3	1 51 .51	5.248	35.1	447
30	31.9	2 44 .00	5.245	31.8	434
40	28.9	3 36 .44	5.244	28.8	436
50	26.1	4 28 .83	5.241	26.1	417
60	23.85	5 21 .27	5.239	23.8	401
70	21.7	6 13 .65	5.237	21.6	393
80	19.9	7 6 .00	5.234	19.8	370
90	18.3	7 58 .30	5.234	18.25	368
100	16.8	8 50 .61	5.232	16.7	361
110	15.5	9 43 .00	5.228	15.5	327
120	14.45	10 35 .37	5.228	14.4	308
156			5.221	11.1	328
191	8.61	16 46 .00			
209			5.222	7.95	219
227	7.22	19 54 .00			
250			5.217	6.50	188
273	5.94	23 54 .00			
306			5.212	5.32	155
339	4.71	29 38 .00			
358			5.211	4.46	134
377	4.20	32 56 .00			
401			5.208	3.94	122
425	3.70	37 6. 00			
449			5.208	3.55	107
473	3.40	41 16 .00			

on the wire remained therefore practically constant. A damping arrangement, immersed in oil, was connected to the brass pin between the two wires. After the zero point had been accurately fixed the torsion head was twisted by steps of 50 degrees, observing at the same time the deflection of the mirror. The torsion head was twisted 450 degrees either way and several complete cycles observed. If Hooke's law holds the deflection of the mirror must be proportional to the twist of the steel wire. Fig. 7 shows that this is the case. The deflections of the mirror are plotted as a function of the actual twist of the platinum-iridium wire, *i. e.*, the angular displacement of the torsion head minus that of the mirror. There

is a slight hysteresis effect. We had to wait nearly half a minute after applying a new twist before the system came to rest. In another experiment we waited three minutes between successive observations, but this had no effect upon the area included between the ascending and descending branches of the curve.

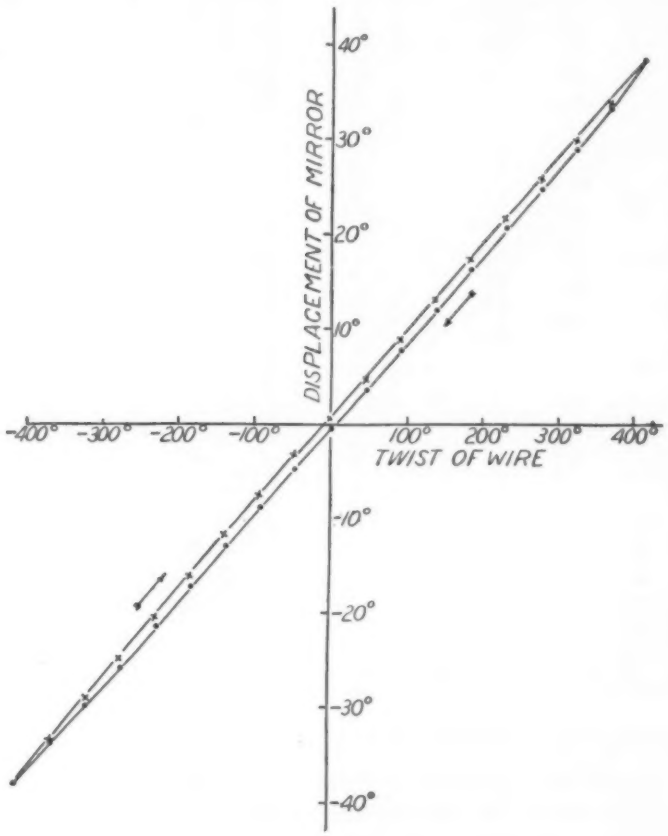


Fig. 7.

It is apparent that for slowly applied twists Hooke's law holds to the same degree of accuracy as for any other wire with small elastic after-effect. This does not exclude the possibility of an elastic after-effect being present in the torsional vibrations of our wire; but if that were the case it must disappear very rapidly. We intend to take up experiments in this direction.

TABLE V.

Same Wire as in Table III. Maximum Amplitude 86.4 *Degrees; Temperature* 24°.1 *C.*

No.	Amplitude.	Coincidences.	Period.	Corr. Ampl.	Log. Dec.
0	83.1	0′ 30″.36			
5	78.1	0 56 .81	5.302	77.6	.00545
10	73.3	1 23 .38	5.299	73.0	
15	68.8	1 49 .83	5.298	68.5	542
20	64.7	2 16 .34	5.295	64.4	
25	60.7	2 42 .75	5.293	60.4	550
30	57.0	3 9 .29	5.290	56.6	
35	53.4	3 35 .65	5.288	53.2	562
40	50.2	4 2 .14	5.285	49.8	571
50	44.0	4 54 .96	5.282	43.8	556
60	38.6	5 47 .71	5.278	38.5	533
70	34.0	6 40 .55	5.275	34.2	509
80	30.2	7 32 .25	5.270	30.4	517
90	26.9	8 25 .94	5.262	26.8	527
100	23.8	9 18 .48	5.255	23.8	512
110	21.1	10 11 .01	5.250	20.9	508
120	18.8	11 3 .44	5.247	18.7	
149			5.238	12.8	512
177	9.6	16 2 .00			
199			5.227	8.35	319
221	6.95	19 52 .00			
239			5.222	6.39	253
257	5.67	23 0 .00			
280			5.217	5.17	202
302	4.6	27 0 .00			
344			5.214	4.2	85
386	3.93	34 18 .00			

Knowing the torsional period of the iron wire with a known moment of inertia, and taking into account the relative angular twists of the two wires, the torsional moment of our wire and the half period with a given moment of inertia may be calculated assuming the elastic laws to hold. Thus we calculated for the case tabulated under 1C a period of 5.375 seconds. The smallest actual period is 6.14 seconds.

10. *Influence of the Amount of Iridium.*—The earlier experiments indicated a large influence of the chemical composition of the wire. This became very apparent in the following series of experiments. We had received three wires, all marked 40 per cent., of different

TABLE VI.

Same Wire as in Table III. Maximum Amplitude 117 Degrees; Temperature 24°.5 C.

No.	Amplitude.	Coincidences.	Period.	Corr. Ampl.	Log. Dec.
0	111.3	0′ 53″.71			
5	104.7	1 20 .29	5.327	104.3	.00526
10	98.6	1 46 .98	5.328	98.2	
15	92.6	2 13 .59	5.327	92.2	538
20	87.1	2 40 .27	5.325	86.5	
25	81.6	3 6 .84	5.324	81.1	565
30	76.5	3 33 .52	5.325	76.0	
35	71.5	4 0 .13	5.321	71.0	569
40	67.1	4 26 .71	5.319	66.7	
45	62.7	4 53 .25	5.317	62.3	618
50	58.2	5 19 .98	5.314	58.0	
55	54.6	5 46 .42	5.310	54.3	565
60	51.1	6 13 .03	5.309	50.7	
65	47.6	6 39 .53	5.309	47.2	620
70	44.3	7 6 .09	5.303	44.0	
75	41.4	7 32 .54	5.303	41.0	643
80	38.2	7 59 .06	5.299	38.0	
85	35.4	8 25 .60	5.298	35.1	649
90	32.9	8 52 .08	5.294	32.7	
95	30.5	9 18 .54	5.291	30.3	647
100	28.35	9 45 .00	5.289	28.1	
110	24.3	10 37 .84	5.279	24.0	666
120	20.9	11 30 .61	5.273	20.7	647
130	18.5	12 23 .25	5.269	17.8	626
140	15.7	13 15 .88	5.263	15.6	577
150	13.75	14 8 .60	5.257	13.6	566
160	12.1	15 1 .12			
180			5.241	9.48	566
201	7.25	18 36 .00			
227			5.231	6.15	316
253	5.00	23 8 .00			
275			5.227	4.42	267
297	3.84	26 58 .00			
315			5.222	3.55	203
333	3.26	30 6 .00			
356			5.217	3.02	154
379	2.78	34 6 .00			

diameters. The experiments described under paragraphs 4, 5 and 6 were made with the finest one of these three. The following gives the results with the others.

(*a*) Wire No. 3, marked 40 per cent iridium, diameter 0.306 mm.

Table VII.

Same Wire as in Table III. Maximum Amplitude 168.9 *Degrees; Temperature* 25°.6 *C.*

No.	Amplitude.	Coincidences.	Period.	Corr. Amp.	Log. Dec.
0	143.3	0′ 58″.60			
2	139.6	1 9 .39			
4	135.9	1 20 .19	5.390	134.9	
6	132.4	1 30 .94	5.393	131.5	
8	129.0	1 41 .75	5.393	128.1	
10	125.4	1 52 .45	5.390	124.5	
15	117.0	2 19 .58	5.391	116.1	.00601
20	109.2	2 46 .41	5.388	108.5	
25	101.9	3 13 .51	5.390	101.1	610
30	94.9	3 40 .34	5.387	94.2	
35	88.2	4 7 .38	5.386	87.5	656
40	81.6	4 34 .19	5.385	81.0	
45	75.8	5 1 .17	5.379	75.2	647
50	70.3	5 27 .94	5.378	69.8	
55	65.0	5 54 .95	5.374	64.5	703
60	59.8	6 21 .80	5.373	59.3	
65	55.0	6 48 .67	5.368	54.5	734
70	50.5	7 15 .47	5.366	50.0	
75	46.2	7 42 .34	5.361	45.8	770
80	42.3	8 9 .11	5.355	42.0	
85	38.3	8 35 .89	5.353	38.0	810
90	35.1	9 2 .60	5.348	34.8	
95	32.0	9 29 .42	5.344	31.7	829
100	29.0	9 56 .00	5.339	28.7	843
110	23.8	10 49 .37	5.327	23.6	867
120	19.5	11 42 .65	5.318	19.3	850
130	16.1	12 35 .69	5.308	16.0	823
140	13.35	13 28 .73	5.295	13.2	729
150	11.3	14 21 .67	5.285	11.2	727
160	9.55	15 14 .45	5.279	9.50	670
170	8.3	16 7 .09	5.272	8.25	628
180	7.8	16 59 .89		7.8	
191			5.254	5.96	552
212	4.76	19 48 .00			
244			5.250	3.88	313
276	3.0	25 24 .00			
305			5.241	2.62	208
334	2.23	30 28 .00			
368			5.235	2.00	148
402	1.77	36 24 .00			

TABLE VIII.

Same Wire as in Table III. Maximum Amplitude 364.3 Degrees; Temperature 24°.6 C.

No.	Amplitude.	Coincidences.	Period.	Corr. Ampl.	Log. Dec.
0	325.3	0′ 54″.40			
1	316.8				
2	308.8	1 5 .76	5.690	305	
3	300.7	1 11 .33			
4	293.4	1 17 .16	5.689	290	
5	286.1	1 22 .70			01081
6	279.4	1 28 .53	5.687	275.8	.01070
7	272.6	1 33 .97			
8	266.6	1 39 .79	5.687	262.8	.01058
9	259.6	1 45 .47			
10	253.6	1 51 .25	5.687	250.4	.01030
12	242.0	2 2 .65	5.688	329.0	.01031
14	230.3	2 14 .00	5.688		
16	220.2	2 25 .40	5.685		
18	210.0	2 36 .70	5.687	207.6	.01022
20	200.6	2 48 .14	5.685	198.2	.01020
25	178.3		5.682	176.0	.01058
30	157.0	3 44 .95	5.681	154.8	.01135
35	137.4		5.679	135.4	.01178
40	119.4	4 41 .81	5.672	117.8	.01742
45	103.1	5 9 .88	5.659	101.8	.01315
50	88.2	5 38 .46	5.647	87.0	.01411
55	74.5	6 6 .64	5.637	73.3	.01531
60	62.0	6 35 .02	5.625	61.0	.01646
65	51.0	7 2 .98	5.603	50.0	.01743
70	41.5	7 31 .11	5.589	40.8	.01832
75	33.45	7 58 .81	5.565	32.7	.01883
80	26.9	8 26 .83	5.534	26.5	.01859
85	21.8	8 54 .02	5.515	21.3	.01818
90	17.7	9 22 .00	5.490	17.3	.01726
95	14.65	9 49 .23	5.471	14.4	.01563
100	12.35	10 16 .36	5.447	12.2	.01517
105	10.33	10 43 .58	5.439	10.15	.01331
110	9.09	11 10 .69	5.416	8.9	.01137
115	7.95	11 37 .64	5.403	7.8	.01073
120	7.0	12 4 .71	5.393	6.9	.01080
125	6.2	12 31 .59	5.386	6.1	.00992
130	5.65	12 58 .65	5.377	5.55	.00848
163			5.338	4.00	.00561
195	2.44	18 46 .00			
210			5.333	2.22	.00288
225	2.00	21 26 .00			
247			5.318	1.80	.00220
269	1.60	25 20 .00			

TABLE IX.

Twist of Torsion Head.	Twist of Mirror.	Differences.	Twist of Torsion Head.	Twist of Mirror.	Differences.
+450	+38.16	+411.84	−450	−38.06	−411.94
400	33.52	366.48	400	33.51	366.49
350	29.19	320.81	350	29.10	320.9
300	24.94	275.06	300	24.86	275.14
250	20.76	229.24	250	20.50	229.5
200	16.35	183.65	200	16.13	183.87
150	12.13	137.87	150	11.81	138.19
100	7.85	92.15	100	7.65	92.35
50	3.55	46.45	50	3.54	46.46
0	− 0.51	− 0.51	0	+ 0.52	+ 0.52
− 50	4.68	45.32	+ 50	4.60	45.40
100	8.85	91.15	100	8.81	91.19
150	13.08	136.92	150	13.19	136.81
200	17.30	182.70	200	17.48	182.52
250	21.50	228.50	250	21.74	228.26
300	25.93	274.07	300	25.98	274.02
350	29.87	320.13	350	30.0	320.0
400	33.95	366.05	400	34.09	365.91
450	38.06	411.94	450	38.16	411.84

This wire was treated in the same manner as the thinner one, *i. e.*, it was heated under load by a current sufficiently large to raise its temperature to a bright yellow heat. Much to our surprise the wire showed only a small decrease in period, starting with 3.336 seconds at 752 degrees, decreasing to 3.330, at 150 degrees, from which point on the decrease was more rapid, down to 3.289 sec. at an amplitude of 2 degrees. The whole change was only 1.5 per cent. as compared with 7 per cent. to 10 per cent. in the thinner wire.

We drew this wire to a smaller diameter without changing its elastic behavior. Before every experiment the wire was again annealed. The results obtained, after the diameter was reduced to 0.25 mm., are given in Table X., under B. The length of the wire was 82.9 cm. In order to allow an easy comparison between all tests made in this part of our investigation all observations were reduced to what they would have been if the wire had had a length of 100 cm. and the moment of inertia such a value so as to make the smallest period just 5 seconds. This was done by multiplying the

amplitudes by the ratio of 100 to the length of the wire in centimeters, and the period by such a factor as to make the period at zero amplitude 5 seconds. The values under A in Table X. are calculated from the experiment tabulated under 1C, in Table I.

TABLE X.

A			*B*		
No.	Amplitude.	Period.	No.	Amplitude.	Period.
0	923	.350	11	936	5.070
2	856	.349	21	904	.069
4	790	.346	40	847	.069
6	728	.343	58	796	.070
8	672	.338	97	(750)	.070
10	612	.332	179	632	.072
15	491	.315	199	474	.072
20	389	.291	239	405	.070
25	300	.268	279	344	.070
30	233	.244	350	(251)	.067
35	178	.218	374	(225)	.066
40	137	.193	437	(162)	.062
45	103	.169	472	(127)	.059
50	84.6	.146	502	98.2	.055
60	52.2	.116	541	73.2	.051
70	39.8	.094	580	54.3	.044
80	29.8	.080	625	38.8	.038
90	24.0	.072	686	26.05	.028
100	19.0	.063	766	16.47	.021
110	15.4	.055	820	12.73	.018
120	12.6	.050	1001	6.14	.010
130	10.3	.040	1150	3.78	.006
140	8.5	.035	1461	1.86	.001
150	6.97	.030			

(*b*) Wire No. 4, marked 40 per cent. iridium, diameter 0.247 mm. This wire was used in the form in which it was received, *i. e.*, not annealed. Now the characteristic peculiarities were more pronounced than with the preceding wire, though considerably less than with the finest wire. After annealing the changes of the period with amplitude were even smaller than with wire No. 3. Table XI. contains the results, C for the unannealed, C′ for the annealed wire.

(*c*) 35 per cent. platinum-iridium wire. The wire had a diameter of 0.20 mm.; it was used as well in the unannealed as in the annealed state. The period remained almost constant from an am-

plitude of 700 degrees down to 100 degrees, after which a small decrease of period with amplitude can be observed, less pronounced in the annealed than in the unannealed state. In both cases it is very small. Table XII. contains the results for the unannealed (D) and for the annealed (D′) wire. Fig. 8 contains the curves for the data given in Tables X.–XII.

The influence of the amount of iridium in the wire is apparent. The 35 per cent. wire shows the characteristic properties to a slight degree. They become more pronounced as the percentage of iridium

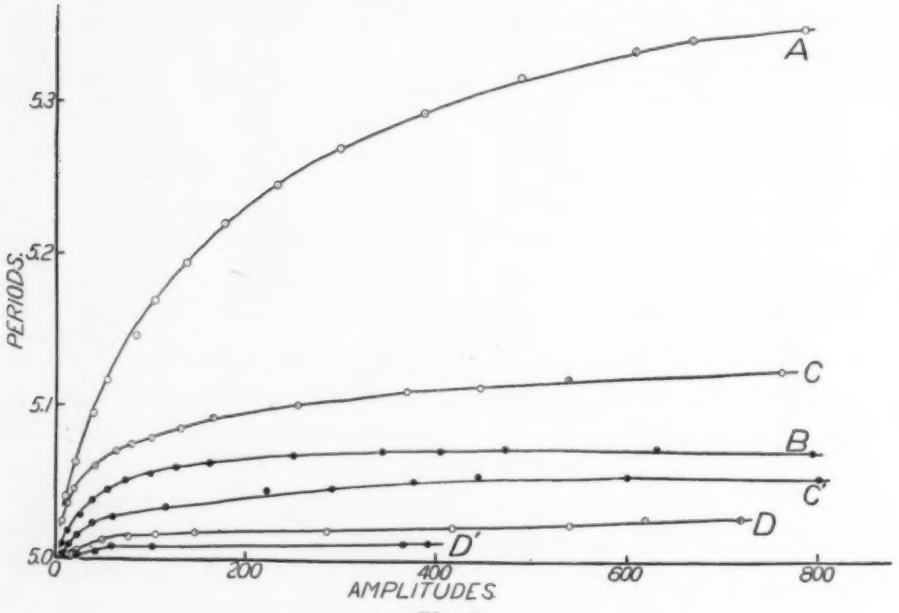


Fig. 8.

increases. The great difference between the thicker wires and the thinner one in our set of 40 per cent. wires, and the great similarity between the thinner one and the one originally used by Guthe led us to the belief that the latter two were originally made of the same alloy, while the thicker ones were made of another mixture containing a smaller relative amount of iridium. Upon inquiry Dr. Heraeus stated that this might be possible, but that no record was kept about the wires. It seems very difficult to draw wires containing more than 40 per cent. of iridium since the material becomes

TABLE XI.

C			*C'*		
No.	Amplitude.	Period.	No.	Amplitude.	Period.
20	784	5.123	21	858	5.053
60	540	.117	36	801	.054
80	447	.112	100	(600)	.054
100	370	.109	160	445	.054
120	(310)	.105	180	420	.054
140	(255)	.100	219	378	.050
160	(207)	.096	273	(291)	.046
180	(165)	.092	321	(223)	.044
200	(131)	.084	405	(116)	.033
220	100.1	.078	490	5.59	.027
240	79.0	.074	530	47.19	.025
260	63.7	.070	570	38.13	.023
280	51.4	.065	607	31.44	.019
300	41.4	.059	683	22.21	.015
340	28.2	.053	743	17.66	.011
380	18.9	.045	825	13.10	.009
400	16.2	.042	926	9.70	.006
436	12.4	.036	1028	7.46	.002
475	9.13	.026	1138	5.99	.002
530	6.22	.024	1411	3.48	.001

extremely brittle. The wires are very hard. Successive holes in the draw plate must not differ more than 0.01 mm. and the wire must be annealed after each drawing. We hope to continue our investigation with a new set of wires just received and extend the investigation also to other brittle wires. At the present state of our knowledge about the wires it seems to us too early to discuss the theory of the phenomena observed, or to propose an empirical formula which will more or less accurately fit our curves. Such a formula will be of value only after we have learned how to control the conditions sufficiently so that we may at will reproduce any given curve.

The large logarithmic decrement points towards an energy loss by internal friction. It should, however, be kept in mind that one of us has pointed out that a carbon filament has a very large decrement but that the latter is perfectly constant. In one of the experiments with a carbon filament the amplitude decreased to one tenth of its original value in 176 swings. The elastic after-effect and its

TABLE XII.

D			D'		
No.	Amplitude.	Period.	No.	Amplitude.	Period.
20	939	5.027	20	392	5.011
40	862	.026	40	379	.011
80	(720)	.027	60	365	.010
120	(620)	.026	810	101.3	.008
160	541	.022	1300	58.4	.008
200	475	.021	1620	42.0	.004
240	419	.020	2110	27.9	.002
280	373	.018	2751	17.5	.002
350	(287)	.018	3558	12.3	.001
420	(230)	.017	4215	8.5	.001
564	(147)	.017			
700	105	.016			
820	76.9	.014			
1000	49.67	.012			
1112	38.55	.007			
1393	21.48	.005			
1573	15.65	.004			
1775	11.58	.003			
2256	6.58	.001			

possible influence upon the period has been discussed above. The behavior of platinum-iridium wire may not be exceptional. Bouasse's work and that of many others point towards a more or less marked deviation from the properties of a perfectly elastic body in all cases; but so far only very small deviations were known. The peculiarities of the platinum-iridium wire are easily studied and may possibly facilitate the understanding of the actual conditions of elastic deformations of ordinary wires.

SUMMARY.

1. Platinum-iridium wires containing over 30 per cent. iridium show striking peculiarities in their elastic behavior which become the more marked the higher the percentage of iridium.

2. For small amplitudes the period of torsional vibration is nearly proportional to the amplitude; but for larger amplitudes the rate of increase of the period becomes smaller and for very large amplitudes the period tends towards a constant value.

3. The logarithmic decrement increases with the amplitude,

reaches a maximum, after which it again decreases and probably reaches a minimum for very large amplitudes. The maximum is the more pronounced the larger the original distortion.

4. If the wire has been allowed to rest some time before the torsional vibrations are produced the total number of vibrations necessary to bring the system to rest is the smaller the larger the initial amplitude.

5. When tested by the static method the wires closely obey Hooke's law. The torsional moment calculated from the static experiment is larger than the largest torsional moment found by the dynamic method.

"BOUND MASS" AND THE FITZGERALD LORENTZ CONTRACTION.[1]

BY WILL. C. BAKER.

FROM the theory that regards electromagnetic momentum as being due to a mass of ether entrained or "bound" by the moving Faraday tubes it follows that the component momentum (per unit volume) in the direction of the motion of the charged body is given by $4\pi\mu N^2 \sin^2\theta\omega$ and that the component perpendicular to this is $4\pi\mu N^2 \sin\theta\cos\theta\omega$ and is directed towards the line along which the body is moving. In these expressions N is the polarization and θ is the angle between the direction of polarization and that of the velocity ω.[2] In considering the momentum of the entire system we need only consider the component parallel to the direction of motion as the summation of the other component must vanish by symmetry. The present note records a simple method of calculating from this idea expressions for both the transverse and longitudinal masses of the system on the supposition that the whole mass is electrical.

It has been shown that as a charged particle attains a steady velocity, a given tube will leave the position occupied by it (relatively to the body) when at rest and will move up towards the equatorial plane, *i. e.*, the plane through the particle normal to the direction of motion.[3] In the case of a body moving along the Z axis with a speed ω the transformation from the configuration that obtains at rest, to that obtaining during steady motion is given (relatively to axes through the particle and moving with it) by

$$x' = x, \quad y' = y, \quad z' \frac{V}{(V^2 - \omega^2)^{\frac{1}{2}}} = z,$$

[1] A paper read at the Boston meeting of the American Physical Society, Dec. 29, 1909.

[2] J. J. Thomson, Electricity and Matter, p. 37.

[3] Heaviside, Phil. Mag., April, 1889. See also J. J. Thomson, Recent Researches, p. 19.

where the unprimed letters refer to a particular part of a tube when at rest and the primed letters refer to the same part of the same tube when in steady motion. V of course is 3×10^{10}. This transformation is evidently that of the FitzGerald-Lorentz contraction,[1] for where $\beta = \omega / V$

$$\frac{V}{(V^2 - \omega^2)^{\frac{1}{2}}} = \frac{1}{(1-\beta^2)^{\frac{1}{2}}}.$$

Taking now these two ideas and regarding the mass of the whole system as being due to the effective part of the "bound" ether, *i. e.*, assuming the whole effective mass as electrical, the Lorentz expression for the ratio of the transverse masses at different velocities follows almost immediately.

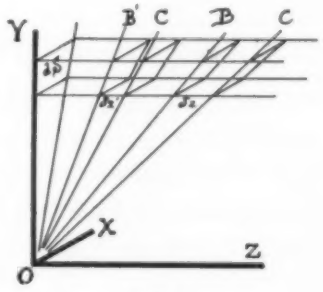


Fig. 1.

Consider axes as shown in the figure, moving with the charged particle as origin. Let BOC represent the postion of an elementary cone of n tubes when the system is moving with a *very small velocity* ω_0. As the velocity increases to a large value, ω', these same tubes will draw up into a position such as $B'OC'$, the change being given by

$$x' = x, \quad y' = y, \quad z' = z(1 - \beta^2)^{\frac{1}{2}}.$$

Now the volumes cut out of these cones by a column parallel to the Z axis and of constant section dS will be "corresponding volumes," *i. e.*, the tubes that occupy one when at rest will occupy the other during the steady motion we are considering. In virtue of the contraction we know that a length δz will shorten to $\delta z(1 - \beta^2)^{\frac{1}{2}}$ also that the elemenatry volumes δv and $\delta v'$ will be given by

$$\delta v = \delta x \delta y \delta z \quad \text{and} \quad \delta v' = \delta x \delta y \delta z (1 - \beta^2)^{\frac{1}{2}}.$$

[1] It is to be noted that this transformation is a direct result of Maxwell's equations. It gives the contraction for the field of the particle independently of the considerations on which the "Fitzgerald Lorentz assumption" is usually based, and it is equally independent of any relativity principle.

The polarizations in the two cases are

$$N = \frac{n}{\delta x \delta z \sin\theta} \quad \text{and} \quad N' = \frac{n}{\delta x \delta z (1-\beta^2)^{\frac{1}{2}} \sin\theta'}.$$

Therefore the Z-components of the momenta *per unit volume* are ($G_z = 4\pi\mu N^2 \sin^2\theta\omega$):

$$G_z = 4\pi\mu \frac{n^2}{\delta x^2 \delta z^2}\omega_0 \quad \text{and} \quad G_z' = 4\pi\mu \frac{n^2}{\delta x^2 \delta z^2 (1-\beta^2)}\omega',$$

and the momenta *in the corresponding volumes* will be

$$g_z = 4\pi\mu \frac{n^2\omega_0}{\delta x^2 \delta z^2}\delta x \delta y \delta z \quad \text{and} \quad g_z' = 4\pi\mu \frac{n^2\omega'}{\delta x^2 \delta z^2 (1-\beta^2)}\delta x \delta y \delta z (1-\beta^2)^{\frac{1}{2}}.$$

Now the transverse mass of a system is given by dividing its (longitudinal) momentum by its (longitudinal) velocity,[1] so it follows at once from the values of g and g' that the transverse masses of the corresponding volumes and (as the ratio is the same for all such pairs) the transverse masses of the entire corresponding systems are in the ratio

$$\frac{m'}{m_0} = (1-\beta^2)^{-\frac{1}{2}}.$$

This is precisely the relation given by Lorentz and verified experimentally by Bucherer[2] in the case of the high velocity particles from radium.

Again, if a force act on the system so as to increase ω, it produces

[1] For let the system be moving with a velocity v through a field of H lines. It will suffer in time δt a change of direction $\delta\theta$. Let the (longitudinal) momentum be G. Now equating impulses

$$Hev\delta t = G\delta\theta;$$

but, where ρ is the radius of curvature of the path,

$$\delta\theta = \frac{\delta s}{\rho} = \frac{v\delta t}{\rho}.$$

Again v^2/ρ is the acceleration normal to the path, so if m_t be put for the *transverse* mass we obtain

$$\frac{m_t v^2}{\rho} = Hve.$$

and on comparing these expressions for $He\rho$ it follows that $M = G/v$. See J J Thomson, Conduction of Electricity through Gases, p. 652.

[2] Annalen der Physik, No. 3, 1909, p. 525.

a change of momentum at a rate given by

$$F = \frac{dG}{dt} = \frac{dG}{d\omega} \cdot \frac{d\omega}{dt} = \frac{1}{V} \frac{dG}{d\beta} \cdot \frac{d\omega}{dt},$$

for $\beta = \omega / V$. Hence the longitudinal mass is

$$m_2' = \frac{1}{V} \cdot \frac{dg_z'}{d\beta}.^1$$

Applying this to our expressions for g_z and g_z', we find that the longitudinal masses are in the ratio

$$\frac{m_2'}{m_2^0} = \frac{1}{(1-\beta^2)^{\frac{1}{2}}}.$$

This is also the relation obtained by Lorentz from the integration of the energy flux.

It is to be noted that the above given expressions refer to the electrical part of the mass only; but this from Bucherer's work would seem to be the whole mass. Also it must be remembered that the Fitzgerald Lorentz contraction — in the above — applies to the tubes of force, *i. e.*, to the field of the charged particle and so through this to actual systems built up of such particles. These considerations show that the question of the deformability of the electron itself tends to pass away and we are led to the conception of the electron advocated by Sir J. J. Thomson; *i. e.*, a simple point charge, the whole inertia of which is due to the entrained ether.[2]

PHYSICAL LABORATORY, SCHOOL OF MINING,
QUEEN'S UNIVERSITY, Kingston, Ont., December 9, 1909.

[1] H. A. Lorentz, Theory of Electrons, p. 37.
[2] Conduction through Gases, pp. 654–655.

THE SECONDARY RADIATION PRODUCED BY THE BETA RAYS OF RADIUM.

BY W. H. BRAGG.

IN the September number of the PHYSICAL REVIEW Mr. S. J. Allen discusses the secondary radiation due to the β rays of radium, and in the introduction to his paper he describes the two theories of secondary radiation due to γ and x rays, which, he says, "are at the present time in prominence."

For one of these I am responsible, and therefore I would ask you leave to correct Mr. Allen's statement of it. On page 178 Mr. Allen writes as follows: "He assumes the beam of γ or x rays to be made up of a number of electric doublets travelling at high speed, and each doublet as composed of a negative and positive electron of the same charge and mass . . .," and again, "Bragg has to assume the existence of a positive electron of the same mass as the negative." Mr. Allen subsequently argues that on such a theory the positive electron should be as capable of isolation as the negative; and that since it has not been isolated in the same way, the theory is not justifiable.

The criticism might have been just if I had made any such statements as those with which Mr. Allen credits me. But I have always held that the positive differed from the negative in more than sign. In my first attempt to frame a corpuscular theory of x and γ rays I considered the possibility of a negative electron being associated with an α particle, but subsequent experiments led me to suppose the mass of the positive to be small compared to that of the negative.[1] This is in accordance, I think, with general ideas. If the negative electron derives its mass from the concentration of its charge on a minute surface of about 10^{-13} cm. diameter, and the positive is more diffused, so to speak, it seems reasonable to expect such a difference in mass. The fact that no positive has been iso-

[1] See, for example, Phil. Mag., VI., 16, p. 938.

lated of less than atomic mass is no argument against such a view, for such a positive may contain numbers of negatives all fitted, except one or two, with their positive counterparts. To conclude, my hypothesis is not therefore "contrary to the known experimental facts"; on the other hand, I think it correlates them simply and fairly completely.

I venture to think that Mr. Allen is also in error in another of the main arguments of his paper. "If the radiation from a pure element," he says on page 188, "is a function of the atomic weight, we should expect as a first approximation the radiation of a salt to be a function of the sum of the atomic weights of its chemical constituents, that is, of its molecular weight." Surely we should expect the opposite conclusion from the given premiss. A bundle of incident rays is limited in quantity; those which are scattered by, for example, the Ba in $BaCl_2$ will often be unavailable for scattering by the Cl; the acts of scattering are, to a considerable extent, mutually exclusive; it is not right to add and subtract, as Mr. Allen does, the effects of the various atoms in the molecules. Many of the results which Mr. Allen classifies as anomalous or inconsistent are quite normal when truly considered in relation to the theory that a β particle acts upon an atom in a way which is independent of the neighborhood or association of other atoms. The amount of secondary radiation will not depend on molecular grouping at all, but rather in a general way on the relative numbers of large and small atoms present. It is not wrong that a sugar solution should give much the same amount of secondary radiation as water, nor that LiCl should give more than Li_2CO_3: nor that CH_2O_2 should give more than H_2O.

Nevertheless, there are certain results of Mr. Allen's work which cannot be explained in this way. The most notable is the comparison of the properties of ethylene chloride and ethylene bromide, which shows that the former gives more secondary radiation than the latter, and possesses three times the absorbing power even when the absorption coefficients are measured in terms of thickness and not of weight. This is very remarkable, and at my request certain students of this university, Messrs. Armin, Dixon and Morton, have repeated the experiments. The liquids were prepared

for them by Prof. Julius Cohen, and they used two methods, one the same as Mr. Allen's, the other different in that the liquid was always enclosed between two thin Al sheets so that no vapor could complicate the results. Their measurements on both methods are quite regular and consistent with each other and they find that ethylene chloride does not absorb three times as much as ethylene bromide, but less, distance for distance. Weight for weight the absorption coefficients are practically the same, as would be expected on the theory that atoms act on β rays independently of each other.

They also find that the secondary radiation from ethylene chloride is less than that from ethylene bromide, as also would be expected.

I am at a loss to explain this complete divergence from Mr. Allen's results; I cannot but think that in some way he has made a mistake. I should like to add a few words in relation to the principles of secondary radiation to which Mr. Allen makes frequent reference in his paper. When any one of what may be called the radiant entities, α, β, γ, x or cathode rays passes through an atom, the chance of its being deflected in a given direction is a function of several variables, but for a ray of specified nature and quality falling on a given atom the chance may be represented by the length of a radius vector drawn in that direction. One of the main purposes of present experiment is to determine the curves which are the loci of the extremities of these radii vectores in the multitude of various cases. Madsen has made approximations to some determinations of this kind for β rays.[1] In the case of β rays and a heavy atom like lead, the atom is nearly centric to the curve; in the case of a light atom like aluminium it is very excentric, as might reasonably be expected. This is quite sufficient to account for the observed dependence of secondary radiation on the angle of incidence, and there is no need at present to suppose a true secondary radiation, or a "specular reflection"; indeed this last is inconsistent with the hypothesis that the β ray acts only on one atom at a time.

THE UNIVERSITY OF LEEDS.

[1] Brisbane Report, January, 1909; Phil. Mag., December, 1909.

A CONVENIENT FORM OF QUARTZ TUBE MERCURY LAMP.

By Chas. T. Knipp.

THE mercury arc in a quartz tube forms a convenient source of ultra-violet light. Several forms of tubes are in general use, prominent among them being the tubes made by Heræus and described by Küch and Retschinsky.[1]

Three years ago the writer had occasion to design and set up a mercury arc-in-quartz tube to be used by the biological department in some work upon bacteria. Fig. 1 shows the tube and manner of mounting. The diameter of the main branch AB is 25 mm., and the length of the straight portion 15 cm. It had, as used in Fig. 1, three outlets. The tubes AM_1 and BM_2 are 20 cm. long and 4 mm. internal diameter, while the exhaust nipple mn was somewhat smaller in diameter. The tube was hung on two asbestos covered pegs mounted on a suitable support. Glass extensions M_1E_1 and M_2E_2 were added of sufficient lengths to support the barometric columns AE_1 and BE_2 respectively. Mercury seals insured tight joints at M_1 and M_2. The vessels E_1 and E_2 served also as the leading in terminals to the arc. After exhausting the tube it was sealed off beyond M_3. The arc was started in the usual way—by lifting E_2 slightly until the mercury would flow from B down to A. The mercury levels at A and B were adjusted readily by raising or lowering E_1 and E_2.

This arc was suited only for low voltages and hence gave out comparatively weak ultra-violet radiations. Any attempt to increase the energy consumption would, because of the vapor pressure, depress the mercury electrodes at A and B. In order to obtain a more intense light the shallow vessels E_1 and E_2 were replaced by narrow tubes (about 1.2 cm. in diameter) extending some distance up the barometric legs. This method automatically kept the mer-

[1] Ann. d. Phys., 20, p. 563, 1906.

cury surfaces at A and B at practically constant levels, and admitted of a small range of light intensity. However, another difficulty was encountered, the increased temperature of the arc interfered with the seal at M_3. In the attempt to obtain a light source of much greater intensity the manner of setting up the tube was entirely changed recently.

In the new arrangement the exhaust nipple *mn* in Fig. 1 was fused off near the main tube, and instead connection was made below the mercury seal M_1 as shown in Fig. 2. This exhaust tube should join the barometric leg at a point p far enough down to

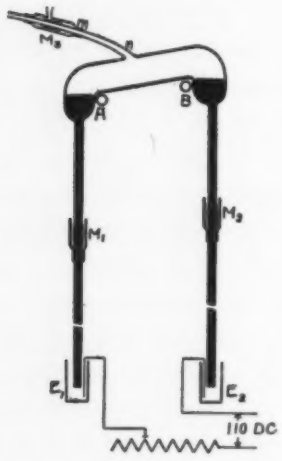


Fig. 1.

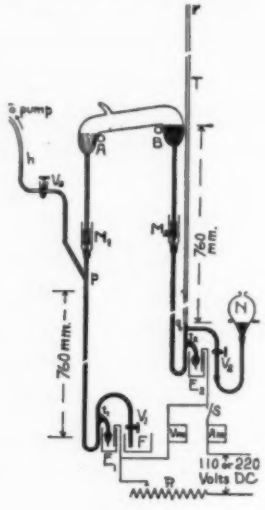


Fig. 2.

admit of lowering the seal M_1. A valve protected by a mercury seal is placed at V_3. The lower end of the leg is shaped in the form of an S having a valve V_1 in its outer right-hand limb and a no. 24 Pt terminal wire sealed into a projection from its middle limb. In the figure the terminal E_1 is shown in the plane of the paper, while in actual construction it is more conveniently joined to the front. This allows the S to be made more compactly than shown. The opening t_1 to the projection carrying the terminal E_1 should be rather large, otherwise some difficulty may be experienced in filling it with mercury. The length of the leg M_1E_1 must be such as to support a barometric column *below* the point p and thus not interfere with the process of exhaustion.

The leg M_2E_2 as will be seen by the figure is similar to the other with the following exceptions: The point q is 760 mm. below the mercury level at B. At q is joined a manometer tube about 150 cm. long and 3 mm. internal bore with its upper end r open. A mercury reservoir N is joined by a short rubber tube to the outlet below the valve V_1 as shown in the figure.

To operate: Fill the cups shown at the terminals E_1 and E_2 with mercury and connect the same through a suitable resistance R and switch S to a 110-volt, or in case very intense light is desired, 220-volt D.C. source. Include an ammeter and voltmeter in the circuit. Now close the valve V_1 and open the valves V_2 and V_3. Let the outlet below V_1 dip into a vessel F containing mercury. Pour mercury into the reservoir N until the mercury rises above the valve V_2 and then close it. Close r with a rubber tube and pinch-cock, and then start the pump. After a few strokes close V_3 and open the valves V_1 and V_2 slightly. This serves to fill the projections at E_1 and E_2. Repeat the operation if necessary. Now open V_1 and V_2, pour a considerable quantity of mercury into N, open the pinch-cock at r, start the pump and then open the valve V_3. After continued pumping the mercury should stand below p in the leg AE_1 and at B in the leg BE_2. When the tube AB, upon sparking, is found evacuated sufficiently (and it is well to let the apparatus stand over night before sealing off) close V_1 and pour mercury slowly into N. The mercury will overflow at B and fill the leg AE_1 rising in both branches and "seal off" the system from the pump. When the mercury has reached h in the exhaust tube close V_3, and when it has reached A in the main tube close V_2. The quartz tube is now ready to start the arc. Close the switch S and open the valve V_2 only long enough to let a stream of mercury flow from B to A. If properly exhausted the tube should light up at once. As the arc burns the current fails off in value. By keeping the current constant—increasing the impressed voltage—the arc will exhibit the characteristics as described by Küch and Retschinsky for increasing vapor pressures. The pressure is read by the manometer qr and may exceed with safety one atmosphere.

The quartz tube AB may have almost any shape which the needs of the investigation may require, keeping in mind the manner

of starting the arc. The electrode B should be comparatively large in area so that a slight depression of the mercury will correspond to a considerable height in the manometer tube qr. Either A or B may be the anode. As A fills up, by condensation and also by overflow from B, it may be lowered at will by opening V_1 for a moment. In case the vacuum falls the tube can be exhausted readily again. The valves should be worked dry to avoid contaminating the mercury. To give added strength to the joints at M_1 and M_2, the pure gum tubing used should be wound over with one layer of adhesive tape which in turn should be wrapped with strong thread. The conduction of heat from the electrodes to these joints is not sufficient to injure them even for continued running at high pressures.

Some of the advantages of this form of quartz tube mercury lamp are: Simplicity of the quartz tube; ease of starting the arc and adjustability of the mercury surfaces at A and B; apparatus may be taken apart readily for purposes of cleaning or for introducing different materials into the arc; when properly mounted it is quite portable.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS

NOTE ON THE REFLECTING POWER OF TANTALUM, TUNGSTEN, AND MOLYBDENUM.

By W. W. Coblentz.

TO many of us it seems sufficient to explain the high luminous efficiency of the new, metal filament, incandescent lamps by saying that is is due to selective emission. The question of the region of the spectrum in which this selectivity occurs seems to be overlooked. In a recent number of the Bulletin of the Bureau of Standards, the writer has given reasons for believing that in these metals the selectivity must fall in the region of the spectrum of wave-lengths less than 1.5 to 2μ and especially in the visible and ultra-violet; for a low reflectivity (and hence high emissivity) in the visible and ultra-violet is a characteristic of pure metals. In other words, since the reflectivity of metals is a smooth and continuous function of the wave-length in the infra-red, there is no reason for expecting these unexamined metals (tungsten, tantalum, etc.) to behave otherwise. Their spectral energy curves must therefore be smooth and continuous in the infra-red (no indentations or protuberances) and experimental curves showing such indentations are to be regarded with suspicion as to the accuracy of the observations.

It is difficult to show experimentally this selective emission in other than "colored metals" (*e. g.*, gold and copper) by means of emission spectra, and since the emissivity in the short wave-lengths appears to have only a small temperature coefficient it is possible to gain some knowledge of the spectral emissivity of the metal at high temperatures by determining its reflectivity at low temperatures. It is therefore proposed to study the reflecting power of the various metals used in incandescent lamps, and as a preliminary the following observations are presented.

To obtain the reflectivity, a vacuum bolometer, mirror spectrometer and fluorite prism are used. The metal, in the form of a

plane surface, is compared with puresi lver of known reflectivity, from which the absolute reflecting power of the unknown metal is readily obtained.

The sample of tantalum (dimensions, 10 × 15 × 1 mm.) was the purest obtainable, and in spite of the excellent polish it shows an excessively low reflectivity throughout the spectrum, as compared with pure metals. The curve in Fig. 1 represents three series of observations, using different adjustments and an angle of incidence on the mirrors of 12 to 15°, the observations usually agreeing to 1 part in 300. In the visible spectrum the reflectivity is only

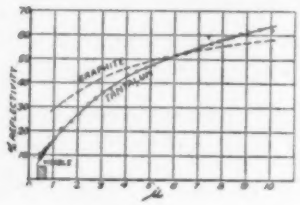


Fig. 1.

10 per cent., which gradually increases to 63 per cent. at 10μ. There is no indication of an approach to constant reflectivity in the infra-red such as obtains in pure metals. In this respect the tantalum now obtainable is not unlike steel, except that the latter has a higher reflectivity (60 per cent. in visible to 95 per cent. at 12μ) throughout the spectrum. The reflectivity is very closely the same as natural graphite (dotted curve, Fig. 1) which increases from a reflectivity of 30 per cent. at 1μ to 58 per cent. at 10μ. The reflection spectrum of tantalum has the characteristic of an impure metal or an alloy. To the eye, the sample appears flawless, so that the low reflectivity in the short wave-lengths cannot be explained on the hypothesis of roughness and scattering of the light. A rough comparison was made with a small spectrophotometer in the visible and violet, which showed that the reflectivity of tantalum continues to decrease toward the violet but not very rapidly as compared with silver.

The low reflectivity in the visible spectrum tends to produce a high emissivity, but contrary to the properties of pure metals, *e. g.*, platinum, the reflectivity continues low (emissivity high) in

the infra-red, which tends to lower the luminous efficiency. Omitting the question of operating temperatures, the reflectivity data give us some clue to the anomalous position of tantalum in the scale of luminous efficiencies of metals. Incandescent lamps of tungsten and osmium have an efficiency of about 1.25 w.p.c. while for no particular reason tantalum must be classified with graphitized carbon with an efficiency of about 2 to 2.5 w.p.c.

The foregoing data on the reflectivity of tantalum and graphite show that this classification is consistent. The radiation constants of these two substances are almost the same (total radiation; tantalum proportional to the 5.3 power of temp., graphitized filament proportional to the 5.1 power of temp.) and the manner in which they differ is in the right direction, since graphite is a non-metal. Hence, while it was somewhat contrary to expectation to find such a low reflectivity in the infra-red, for tantalum, this is not inconsistent with other data. Since the reflectivity curve is uniform it is evident that the spectral energy curve must also be smooth and free from such indentations as have been found in the emission spectra of non-metals.

The reflecting power of tungsten and of molybdenum is entirely different from tantalum. These two metals have a low reflectivity (50 p.c. and 48 p.c. respectively) in the yellow, which rises abruptly to 85 p.c. at 2 μ, beyond which point the reflecting power increases gradually to 96 p.c. at 10 μ. This is a characteristic of pure metals.

Since molybdenum and tungsten have almost the same reflectivity throughout the spectrum their luminous efficiencies must be practically the same, when operated at the same temperature; and since the former is tough and ductile, while the latter is brittle, it would be advantageous to use the former as a lamp filament, provided certain other physical defects can be overcome.

WASHINGTON, D. C.,
March 1, 1910.

THE CATHODE EQUILIBRIUM IN THE WESTON STANDARD CELL.

BY G. A. HULETT.

IN previous articles[1] the cathode systems of standard cells have been studied from the chemical and electrochemical standpoints. The cathode system of the cadmium or Weston cell is made from a saturated solution of cadmium sulphate to which is added an excess of cadmium sulphate crystals, mercurous sulphate and mercury. In this system it was found that the depolarizer, mercurous sulphate, was hydrolyzed by the cadmium sulphate solution but the reaction proved to be so very slow that it was not possible to bring the system to equilibrium by rotating it at 25°. The various facts obtained in these experiments were best explained by assuming that this hydrolysis proceeded more rapidly at the surface of the mercury than at any other point in the system. This assumption has been objected to by Dr. F. A. Wolff as not in accord with some of his experiments,[2] and, as this assumption is also a part of the explanation of the slow changes which have been observed in the Weston standard cells, it seemed to be desirable to get more evidence on the point.

In the previous work a tube, 32 × 150 mm., was charged with mercury, mercurous sulphate, cadmium sulphate and solution in about the proportions used in standard cells. It was so arranged that the anode leg could be removed, the tube closed and rotated horizonally in a constant temperature bath and whenever desired the anode was adjusted and the value of the cell determined.[3] It was found that the rotation of the cathode system of this cell increased its E.M.F. and in most experiments by about two millivolts. This increase in the value of the cell indicated an increase in the concentration of the mercury in the solution of the cathode

[1] Zeit. Phys. Chem., 49, 483. Trans. Amer. Electrochemical Soc., 5, 59. PHYS. REV., 22, 321; 23, 166; 25, 16 and 27, 337.

[2] PHYS. REV., 28, 310.

[3] PHYS. REV., 27, 346.

system, a conclusion which has been fully substantiated by analyses of these solutions.[1] When the system was allowed to stand, after rotation, the E.M.F. was found to slowly decrease, but there was not a corresponding decrease in the mercury concentration in the solution above the paste or in the paste, so that it was evident that some change had taken place which decreased the mercury concentration in the solution which formed the surface layer between the paste and the mercury electrode. This conclusion was arrived at so many times and under such varying conditions that there could be no question as to facts. At first it seemed that there must be some secondary reaction which caused the decrease at the boundary,[2] but when more information was obtained it was seen that this change could be readily explained as the result of an increase of the rate of hydrolysis at the mercury surface, for that would mean an increase in the basic salt and acid at the expense of the normal salt, a condition which would decrease the mercury concentration in this surface next to the electrode and it is the mercury concentration in this layer alone which determines the cathode potential of the cell. It was also noted that the acid formed by the hydrolysis could diffuse out to regions where there was little or no acid and thus the reaction would proceed until nothing but the basic salt was left in the surface next to the mercury cathode. The basic salt has only a very small solubility and such a condition would cause the E.M.F. of the cell to decrease to an abnormally low value, and this is exactly what has happened in a number of cases[3] and with some of our own cells.

Now the question as to whether the rate of hydrolysis of mercurous sulphate is affected by the presence or absence of mercury in this system can be readily tested as follows: If two rotation experiments are carried out, alike in all respects except mercury is absent from one tube, there should be a difference in the rate and, since none of these systems can be brought to equilibrium in any experimental time, there should also be a difference in the amount of the hydrolysis in the two cases.

[1] PHYS. REV., 27, 348.
[2] PHYS. REV., 25, 27.
[3] Philos. Trans., A, 207, 400.

The same apparatus[1] which had served for the previous work was used for this test. The cathode tube was charged with the normal amount of mercurous sulphate, cadmium sulphate and saturated solution but precautions were taken to exclude mercury. The mercurous sulphate was prepared chemically.[2] It was white and crystalline and easily freed from mercury by decantation, using a 1 to 6 sulphuric acid; it was then washed with alcohol and a cadmium sulphate solution. The tube was rotated as previously described; from time to time some of the clear liquid was removed and tested on a clean mercury surface against a cadmium amalgam, thus forming with it a standard cell.[3] In this way the following results were obtained:

Time of Rotation.	E.M.F.
23½ hours.	1.01800
3 days.	50
7	66
12	70

On continuing the rotation no further change was observed, while in the previous experiments which were like this one, except that mercury was present, values ranging about 1.020 volts were obtained before a steady state was reached. It was of interest now to determine the effect of adding mercury to this particular system, accordingly 20 c.c. of the clear liquid was removed and an equal volume of mercury added. An *increase* in mercury concentration was at once indicated by the E.M.F. measurements as the following results show:

Time of Rotation after Adding the Mercury.	E.M.F.
7 hours.	1.01895
2 days.	1.01925
9	1.01945
18	1.01955

A second experiment was tried in which electrolytic mercurous sulphate was used and before mercury was added to the cathode system the following results were obtained:

[1] PHYS. REV., 27, 346.
[2] PHYS. REV., 127, 345.
[3] PHYS. REV., 25, 21.

Time of Rotation.	E.M.F.
$3\frac{1}{2}$ hours.	1.01825
2 days.	68
5	80
10	90
22	1.01910

Mercury was now added to the system and the rotation continued.

Time of Rotation.	E.M.F.
2 days.	1.01924
9	50

It was not possible to remove the mercury from the electrolytic mercury as completely as in the case of the chemically prepared depolarizer, but the effect of the presence of an excess of mercury was still distinctly measurable. The increase in the E.M.F. of the cells show a corresponding increase in the mercury concentration of the cathode system as has been abundantly shown so these experiments give good evidence of the effect of mercury in increasing the rate of hydrolysis of mercurous sulphate in a cadmium sulphate solution.

In the rotation experiments the cathode tube with the adjustable anode contained materials in the same proportions as an ordinary Weston standard cell and they have shown the same values as the regular cells made from the same materials. When, however, the cathode was rotated the possible changes were made to take place more rapidly and could then be readily studied. These changes are such as would take place in any Weston standard cell when the cathode system is similarly treated, for example a cell (W. 141) which was made in December, 1906, and has not varied over .00002 V., from 1.01834 at 25° for three years, was opened and the cathode paste carefully but thoroughly stirred. The E.M.F. immediately increased to 1.01881, then it decreased rapidly at first and in two weeks was down to 1.01837 and still going down. All the time this cell had been constantly at 25°. Another cell (W. 38) made in May, 1906, was opened in February, 1907, and the paste stirred. It did not go so high and its value December 25, 1909, was 1.01813, so we get a variety of results from this experiment as might be expected from our present knowledge of the hydrolysis which takes place.

Two cells which had fallen to very low values A17, made in November, 1905, showed the value of 1.01212 on December 25, 1909, and A51 made in July, 1906, was 1.01794, December 25, 1909. These cells were opened and an adjustable anode showed that the anodes of these cells were normal and that the changes had been exclusively in the cathode system. When these cathode systems were stirred the values 1.01922 and 1.02310 respectively were registered and these cells are now showing decreasing values. This is further evidence that in the ordinary cell the surface layer of the electrolyte surrounding the mercury cathode has a distinctly lower mercury concentration than the rest of the electrolyte in the paste and above and that there must have been an increased rate of reaction next to the mercury.

No such changes as have been observed could occur if this system was in equilibrium for the possible disturbances due to surface tension or oxidation have been shown to be without effect in the experiments as carried out[1]. On the other hand, hydrolysis has been shown to take place in this system and it offers an explanation of the observed changes. The hydrolysis, with which we are concerned here, is an interaction between mercurous sulphate and water, which results in the formation of a basic salt and acid according to the following reaction: $2\ Hg_2SO_4 + H_2O = Hg_2(OH)_2Hg_2SO_4 + H_2SO_4$. The paste is made by mixing solid mercurous sulphate, cadmium sulphate and a saturated solution of cadmium sulphate. The mercurous sulphate very rapidly saturates the solution as has been shown, even when large crystals are used,[2] but the hydrolysis is an exceptionally slow reaction and equilibrium is not reached until a certain concentration of sulphuric acid is formed, the equilibrium concentration, and not until this point is reached are further changes excluded. The basic salt formed remains in solution at first and increases the concentration of the mercury and this continues until the solution is saturated in respect to the basic salt which may then appear as a solid phase. When the basic salt appears as a solid phase, the increase in mercury concentration ceases and begins to decrease as the hydrolysis furnishes an increasing acid concen-

[1] PHYS. REV., 25, 21.
[2] PHYS. REV., 27, 360.

tration and the decreasing mercury concentration continues until the acid concentration has reached .08 molecular weight in a liter.[1] Then, and not until then, is this system in equilibrium. The situation may best be illustrated by the following diagram which gives the concentrations of mercurous mercury in a saturated solution of cadmium sulphate as a function of the varying concentration of sulphuric acid, all at 25°.

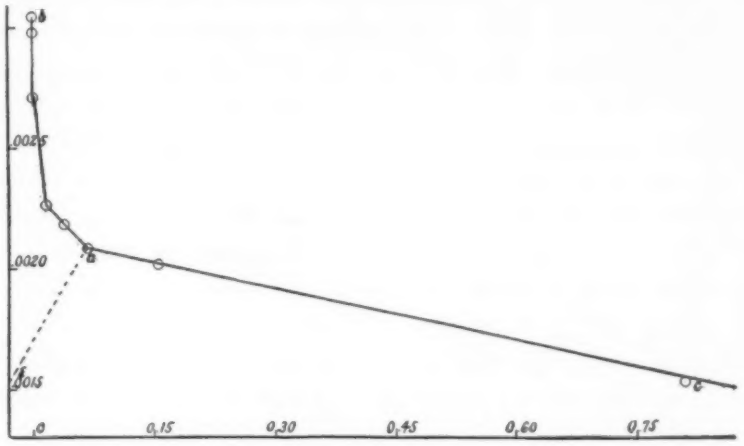


Fig. 1.

Starting at the point *c*, where the cadmium sulphate solution contains .810 molecular weight of sulphuric acid to a liter, there was found to be .001524 molecular weight of mercurous sulphate in solution when the system was in equilibrium.[2] On decreasing the acid concentration the mercurous sulphate concentration increased uniformly to the point *a* where the acid concentration was .08 molecular weight in a liter. This line *ac* is an equilibrium curve for this system and represents the solubility of mercurous sulphate in a saturated solution of cadmium sulphate with varying concentrations of sulphuric acid. This curve extends further to the right but stops at *a*, along this line the system is made up of mercury and the saturated solution with crystallized cadmium sulphate and normal mercurous sulphate as solid phases. The basic salt cannot

[1] PHYS. REV., 27, 353.
[2] PHYS. REV., 27, 349.

exist in this region if the system is in equilibrium. At the point *a* a distinct break occurs in the solubility curve, for on continuing to decrease the acid concentration the curve *ab* was obtained. A break of this kind in a solubility curve shows that some change has taken place in the nature or number of the solid phases. The change in this system was found to be due to the appearance, at this point, of the basic mercurous sulphate so the point *a* gives the acid concentration at which there may exist in equilibrium, mercurous sulphate, basic mercurous sulphate, cadmium sulphate crystals, mercury, saturated solution and vapor. Now these phases may all result from the four components, cadmium sulphate crystals, mercurous sulphate, water and mercury, therefore at the point *a* we are dealing with a nonvariant system and in order to proceed along the equilibrium curve *ac* one phase must disappear and it is the basic salt in this case. Likewise, on taking the system to the left of the point *a*, where the acid concentrations are less than .08, the normal salt must *disappear* if the system is in equilibrium. Now in all the experiments made with the system in this region, the normal salt was always present in large excess, so the curve *ab* could not have been an equilibrium curve, but represented the results obtained by rotating such systems until no further changes were observed. If these systems had come to equilibrium they would all have arrived at the point *a*, since there was always more than enough normal mercurous sulphate present to generate the necessary .08 molecular concentration of sulphuric acid The reason they did not come to equilibrium is due to the slow rate of hydrolysis. When we do have the system in equilibrium in this region—from zero to .08 molecular concentration of sulphuric acid—only the basic mercurous sulphate can be present, and since this has only a small solubility the equilibrium curve here would be something like the dotted line *ae* which would represent the solubility of basic mercurous sulphate in a saturated cadmium sulphate solution with varying concentrations of sulphuric acid.

If we cannot bring the cathode system of the Weston standard cell to equilibrium by rotating it, there is of course no chance of attaining equilibrium in the cell where the conditions are most unfavorable, but these unfavorable conditions bring about, in a

relatively short time, a "steady state" which is not noticeably affected by ordinary temperature changes, and the result is that most of the Weston standard cells are serviceable for years. They suggest under-cooled liquids or supersaturated solutions, as now and then a cell will slowly decrease to low and indefinite values, while others made at the same time and in the same way do not. In order to furnish a reliable basis of electromotive force it is necessary to set up these cells at regular intervals and reject those found to be decreasing to low values. When certain precautions are taken the Weston standard cell has a very satisfactory reproducibility, but the construction of these and Clark cells may best be treated in a separate article.

PRINCETON, January, 1910.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE FIFTY-SECOND MEETING.

THE spring meeting of the Physical Society was held in Washington, D. C., on Friday, April 22, and Saturday, April 23, 1910.

At the Friday afternoon session, which was held in the Main Lecture Room of George Washington University, President Henry Crew presided.

The Saturday sessions were held at the Bureau of Standards, Vice-President W. F. Magie presiding during the morning, and Ernest Merritt acting as temporary chairman during the afternoon.

The following papers were presented:

Notes on Potentiometer Design. FRANK WENNER.

The Relation between Zero Shift and Size of Wire in the Moving Coil Galvanometer. WALTER P. WHITE.

A Graphic Recorder for Cooling Curves. C. B. THWING.

The Platinum-Rhodium Thermoelement from 0° to 1755°. ARTHUR L. DAY and ROBERT B. SOSMAN.

Preliminary Report on the Temperature Coefficient of Resistivity of Copper. J. H. DELLINGER.

A Test Object for Photometry by Visual Acuity. HERBERT E. IVES.

The Selective Radiation of Platinum. E. P. HYDE.

Spectral Energy Curves. (By title.) W. W. COBLENTZ.

A Quantitative Study of the Emission of Hydrogen, Argon, and Helium in the Visible Spectrum. P. G. NUTTING and ORIN TUGMAN.

Notes on the Physics of Meteor Crater, Arizona. W. F. MAGIE.

A Modified Method for the Determination of Relative Wave-Lengths, Especially Adapted to the Establishment of Secondary Spectroscopic Standards. I. G. PRIEST.

On the Reflecting Power of Ice in the Extreme Infra-red Spectrum. A. TROWBRIDGE and B. J. SPENCE.

The Selective Reflection of Oxides. L. B. MORSE.

Some Phosphorescent Salts of Sodium with Cadmium. C. W. WAGGONER.

The Distribution of Energy in the Spectrum of Sidot Blende. (By title.) C. A. PIERCE.

Further Experiments on Fluorescence Absorption. E. L. NICHOLS and ERNEST MERRITT.

The Effect of Temperature on the Ionization of a Gas. J. HARRY CLO.

The Isolation of an Ion and a Precision Measurement of its Charge. R. A. MILLIKAN.

A New Form of Rotating Sectored Disk for Spectro-Photometric Measurements. E. P. HYDE.

Neutral Contacts and Switches. WALTER P. WHITE.

A New Form of Surface Bolometer. E. H. NORTHRUP.

The Inductance of a Metal Tube Bent into the Form of a Circular Ring. F. W. GROVER.

Absolute Formula for the Mutual Induction of Coaxial Cylindrical Current Sheets. G. R. OLSHAUSEN.

The Reliability of Magnetic Measurements on Rods. CHAS. W. BURROWS.

Some Investigations on Mica Condensers. H. L. CURTIS.

A Condition Limiting the Precision Attainable by Increasing the "Order," in the "Method of Diameters" for Wave-Lengths. I. G. PRIEST.

A Method for Measuring and Establishing the Ten-to-One Ratio of Resistance. FRANK WENNER.

A Direct Reading Candle Power Scale for Photometers. G. W. MIDDLEKAUFF.

ERNEST MERRITT, *Secretary.*

THE USE OF THE ELECTRICAL RESISTANCE THERMOMETER IN APPARATUS FOR THE ABSOLUTE MEASUREMENT OF SOLAR RADIATION.[1]

BY C. F. MARVIN.

POUILLET'S actinometer, although greatly improved by a long list of physicists has, as yet, failed to give us a satisfactory absolute measurement of solar radiations. This is mostly because mercurial thermometers are employed. The problem is a problem in calorimetry. The use of electrical resistance thermometers removes serious difficulties almost completely, as, for example, bad heat conductivity and losses along the thermometer stem, heterogeneous material of different specific heats, and the uncertain relation between temperature change shown by thermometer and true change in temperature of calorimeter. A calorimeter,

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 21–24, 1909.

or "block," for absorbing solar radiation can be constructed of thin metallic ribbon which is at once a thermometric bulb and a calorimeter. Its specific heat measured with great accuracy *in situ* by sending through the ribbon measured electric currents and observing the heating effects. The details of a pyrheliometer constructed on this plan with accessory apparatus were shown by lantern slides.

NOTE ON THE RELATION BETWEEN THE RESISTANCE OF NICKEL AND THE TEMPERATURE.[1]

BY C. F. MARVIN.

THE use of nickel in the construction of resistance thermometers is not recognized as completely as its merits and advantages warrant. It cannot compete with platinum at high temperatures, but apparently nickel is as good, if not better than platinum at lower ranges of temperature, say below 350° C. The data for several samples of nickel, although admittedly scanty, show that the curve of variation of resistance with temperature is strictly logarithmic, viz.: $\log r = a + mt$. Hence two accurate measurements of resistance at two temperatures suffice to fix the curve. This law of resistance change is of peculiar advantage in resistance measurements with a slide-wire bridge, since readings of the slider on a scale of equal parts can be made to accurately represent temperature over a wider range than otherwise and troublesome computations are avoided. The residuals between the observed and computed values in the data so far available do not exceed 0.1 per cent.

THE EFFECT OF FILTER PAPER UPON THE MASS AND FORM OF THE DEPOSIT IN THE SILVER COULOMETER.[1]

BY E. B. ROSA, G. W. VINAL AND A. S. MCDANIEL.

THE Rayleigh coulometer, which has been widely used and many times investigated in several different countries, has generally been found to yield a heavier deposit of silver than the Kohlrausch or Richards forms in which filter paper is not employed. The difference has been attributed to secondary reactions at the anode, in which a complex ion is formed, and this complex ion being brought to the cathode by convection currents, has been supposed to yield a larger quantity of silver for each univalent charge. We have shown that the trouble with the Rayleigh coulometer is not due to anode products, but to the reduction of neutral silver nitrate by furfuraldehyde resulting from the hydrolysis of the cellulose of the filter paper, the heavier deposit resulting from

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 21–24, 1909.

a certain quantity of colloidal silver that is produced and deposited together with that carried by the current. The colloidal silver not only increases the mass of the deposit by an appreciable amount, but changes the form of the deposit, as seen by the unaided eye and also as seen by a microscope.

A large number of experiments have been made to test the various questions involved, and many photographs aud microphotographs made of the silver deposits.

Some Curious Phenomena Observed in Connection with Melde's Experiment.[1]

By J. S. Stokes.

To one prong of an electrically maintained tuning fork, vibrating in a horizontal plane, at a rate about one hundred times per second was attached a linen thread. This was drawn out at right angles to the prong, passed over a small brass pulley and a weight attached a few inches below the pulley.

When adjustment swere such that the string was vibrating in segments with sharp nodes, the pulley was seen to rotate slowly with its upper edge moving toward the prong.

A little beeswax was now applied to the string in contact with the pulley and the latter now rotated in the opposite direction. A drop of kerosene was then applied to the string in the groove of the pulley and the direction of rotation was again reversed.

The fork was now slipped toward the pulley until the overhanging portion of the string vibrated as a whole, when the speed of rotation of the pulley was increased to more than one hundred revolutions per minute.

The explanations offered for these phenomena were as follows :

When the prong strikes toward the pulley, the slackening of the tension in the string at the prong, sends a wave form along the string, which on reaching the pulley tends to throw the string outward clear of the pulley and to allow the weight to drop without turning the pulley. When the prong strikes backward, the inertia of the weight and the tightening of the string create pressure on the pulley, and the consequent friction rotates it toward the fork.

When wax is applied to the string, the latter clings to the pulley and the descending weight rotates it outwards and the string now seems to exert less friction on the pulley on the backward stroke than on the forward. This is probably analogous to the slipping of the belt on a flywheel of an engine when the wheel is started too suddenly.

[1] Abstract of a paper presented at the Urbana meeting of the Physical Society, November 27, 1909.

When the oil is applied to the string, conditions return to what they were at the start and the wheel reverses direction of motion.

No explanation was offered for the increased speed of the pulley when the overhanging vertical portion of the string was vibrating and the horizontal part seemed not to be doing so. The plane of vibration of this vertical portion has been observed to rotate at about one half the speed of the pulley, without any rotation of the weight it supports. This is no doubt due to a transfer of the twist in the string, along the string.

For the successful performance of the experiment certain conditions must be realized: viz., the pulley must turn easily on its pivots as the torque obtained is very small. The groove must be well polished. The string must not be too large, about no. 30 linen thread works well. The weight must not hang more than a few inches below the pulley. The tension in the string must be such that the string vibrates in two or more segments. The string vibrating as a whole seems to rotate the pulley always in the same direction. Too much wax must not be used on the string. The string can be rubbed with a cloth to remove excess of wax. Also the groove of the pulley. As far as the demonstrator has been able to ascertain, these phenomena have not been hitherto observed or described by any one.

INSULATION OF OBSERVATORY DOMES FOR PROTECTING TELESCOPES AND OTHER PHYSICAL APPARATUS AGAINST UNNECESSARY EXTREMES OF HEAT AND COLD.[1]

BY DAVID TODD.

PRIOR to constructing the domes of the new Observatory at Amherst, thermometric experiments were made with a variety of insulating materials to ascertain which was best adaptable to the necessary conditions. Accordingly the 20-foot dome is insulated with about 3 inches of granulated cork, and the 35-foot dome with 1½ inches of hair felt. The base of each dome is further protected by a sliding diaphragm of felt. This simple arrangement affords great protection to the instruments, reducing the range of temperature variation, and also protecting them in large measure against dust and condensation.

NOTE ON THE COEFFICIENTS OF DIFFUSION OF THE EMANATION AND THE ACTIVE DEPOSIT PARTICLES OF ACTINIUM.[1]

BY PROF. J. C. MCLENNAN.

IN this note attention is drawn to some results recently published by W. T. Kennedy on "The Active Deposits of Actinium." In his experiments it was shown that a maximum activity existed in different gases

[1] Abstract of a paper presented at the Boston meeting of the Physical Society, December 28–31, 1909.

at a definite distance from the salt and the position of this maximum was found to vary with the pressure at which the active deposits were investigated.

The variation in the position of this maximum was ascribed to diffusion, and the relative coefficients of diffusion for this maximum effect in the gases air, hydrogen and carbon dioxide were reduced.

It is now pointed out that these relative coefficients of diffusion are practically identical with those found by Russ for the diffusion of the emanation from actinium. The effect observed by Kennedy, however, involved the diffusion not only of the emanation but also of the active deposit particles, and the results go to show that the active deposit particles and those of the emanation possess the same diffusion coefficient.

In the recent work of Wellish [1] and in that of Franck [2] also it has been shown that ions produced from different substances and differing as widely as 100 per cent. in their mass possess the same mobilities in a gas such as hydrogen. In particular it has been shown that ions produced by Röntgen rays in methyl iodide and also the particles which constitute the active substance thorium D in hydrogen possess the mobilities of hydrogen ions themselves.

This result is in accordance with a theory of ionization recently brought forward by Sutherland [3] in which he draws the conclusion that a viscosity arising from the electric polarization of the neighboring molecules by an ion must exist and exert a predominant influence in determining mobilities.

According to Sutherland's theory this electric viscosity would come into operation in the diffusion of ions also, and it would explain the close agreement found by Kennedy between the coefficients of diffusion of the particles of the active deposits from actinium and that of the particles constituting the emanation of this substance.

A Quantitative Study of the Emission of Hydrogen, Argon and Helium in the Visible Spectrum.[4]

By P. G. Nutting and Orin Tugman.

An ideal gas spectrum would consist of but a single line whose intensity would vary in direct proportion to the internal energy of the gas. Ordinary lines are known to diverge both ways from this linear relation. Primary lines increase rapidly at first and more slowly with intense excitation, secondary lines increase slowly at first and very rapidly later. Some lines (*e. g.*, the blue spectrum of argon) have been supposed to appear only when the discharge through the gas is extremely

[1] Proc. Roy. Soc., A., 557, July 31, 1909.

[2] Verh. der Deut. Phys. Ges., No. 19, 3397, 1909.

[3] Phil. Mag., September, 1909.

[4] Abstract of a paper presented at the Washington meeting of the Physical Society, April 22–23, 1910.

violent. The spectra chosen for study exhibit lines and groups of lines of both these types.

The method of study was to excite the gases in large Plücker tubes by sending a known alternating current through them. A spectrophotometric comparison with a reference tube held at constant current (or pressure) then gave line intensity as a function of current (or pressure). Mean potential gradients were determined subsequently. A special form of tube with porcelain capillary gave more constant results than all-glass tubes and carried twenty times the current (half an ampere) that these would safely carry.

In the hydrogen spectrum three different portions of the primary spectrum, red $\lambda 611$, green $\lambda 545$ and blue $\lambda 460$ gave the same variation with current

$$\frac{P}{P_{20}} = 1.042\,\frac{i}{i_{20}} - .0432\left(\frac{i}{i_{20}}\right)^2 + .00098\left(\frac{i}{i_{20}}\right)^2.$$

The three lines, α, β, γ, studied of the secondary spectrum were found to bear a simple relation $S = P^m$ to the primary where $m_\alpha = 1.67$, $m_\beta = 1.35$, $m_\gamma = 1.14 \pm .02$. m as a function of wave-length may be closely represented by $m - 1 = 681(1/366 - 1/\lambda)$ indicating very slight variability out toward the head of the Balmer series.

Intensity at constant current (20 m.a.) and variable gas density gave a pronounced maximum near 1 mm.

A series spark with condenser gave, with low current, relative intensities of α, β, γ and P, such as were obtained with 250 m.a. of single phase alternating current.

In the argon spectrum the lines studied were red $\lambda 696$, orange $\lambda 603$, green $\lambda 556$, blue $\lambda 454$ and violet $\lambda 420$ in the primary spectrum and the groups at $\lambda 443$ and $\lambda 481$ in the secondary or spark spectrum. Each of these departs widely from a linear relation to current. The secondary lines were followed from 450 m.a. down to 100 m.a., and the extrapolated curves indicate that they persist down to zero current.

The pressure at which the intensity is a maximum is 0.7 for the primary lines and under 0.2 for the secondary. Series spark and condenser gave relative line intensities such as were produced with about 200 m.a. of steady current.

In the helium spectrum no new lines could be brought out with either the heaviest currents or the most violent disruptive discharge available. In all cases the ratio of line intensity to current steadily decreases with increase of current. The curves for the bright red, yellow, blue and violet lines lie close together, but the bright green line lies much higher. The pressure at which the line intensity is a maximum varies from 4 mm. for the extreme red line $\lambda 706$ to 2 mm. for the blue $\lambda 447$. The faint green line $\lambda 504$ apparently has no maximum, its intensity increasing more and more rapidly with decrease in pressure down to 0.5 mm.